Maths Formula Sheet

Chapter by chapter, topic by topic. Written by the PadhoNa team for quick revision.

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CHAPTER 1

Basic Mathematics and Logarithm

Logarithm

DEFINITION

$$\log_a x = y \iff a^y = x, \qquad a > 0,\ a \neq 1,\ x > 0$$

PRODUCT AND QUOTIENT

$$\log_a(mn) = \log_a m + \log_a n, \qquad \log_a \frac{m}{n} = \log_a m - \log_a n$$

POWER

$$\log_a m^k = k \log_a m, \qquad \log_{a^k} m = \frac{1}{k} \log_a m$$

CHANGE OF BASE

$$\log_a m = \frac{\log_b m}{\log_b a}, \qquad \log_a b \cdot \log_b a = 1$$

INVERSE

$$a^{\log_a x} = x, \qquad a^{\log_b c} = c^{\log_b a}$$

INEQUALITIES

$$a > 1 :\ \log_a x > \log_a y \iff x > y > 0; \qquad 0 < a < 1 :\ \log_a x > \log_a y \iff 0 < x < y$$

Inequalities and modulus

MODULUS

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}, \qquad \sqrt{x^2} = |x|$$

MODULUS INEQUALITIES

$$|x| < a \iff -a < x < a, \qquad |x| > a \iff x < -a \text{ or } x > a \quad (a > 0)$$

TRIANGLE INEQUALITY

$$\big||a|-|b|\big| \le |a \pm b| \le |a|+|b|$$

WAVY CURVE METHOD

$$\frac{(x-a_1)(x-a_2)\cdots}{(x-b_1)(x-b_2)\cdots} > 0$$

Mark the roots on a number line; the sign alternates between consecutive roots (a root of even power does not change the sign). Exclude the b_i.

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CHAPTER 2

Quadratic Equations

Roots and coefficients

ROOTS

$$ax^2+bx+c=0 \;\Rightarrow\; x=\frac{-b\pm\sqrt{D}}{2a}, \qquad D=b^2-4ac$$

SUM AND PRODUCT

$$\alpha+\beta=-\frac{b}{a}, \qquad \alpha\beta=\frac{c}{a}, \qquad |\alpha-\beta|=\frac{\sqrt{D}}{|a|}$$

EQUATION FROM ROOTS

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

CUBIC

$$ax^3+bx^2+cx+d=0: \;\sum\alpha=-\frac{b}{a},\ \sum\alpha\beta=\frac{c}{a},\ \alpha\beta\gamma=-\frac{d}{a}$$

Nature of roots

$$D>0:\text{ real, distinct}; \qquad D=0:\text{ real, equal}; \qquad D<0:\text{ non-real (conjugate pair)}$$

RATIONAL COEFFICIENTS

$$D\text{ a perfect square}\Rightarrow\text{rational roots}; \qquad \text{irrational roots occur in pairs } p\pm\sqrt{q}$$

Common roots

ONE COMMON ROOT

$$(c_1a_2-c_2a_1)^2=(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)$$

BOTH ROOTS COMMON

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Location of roots

VERTEX

$$f(x)=ax^2+bx+c:\text{ vertex at } x=-\frac{b}{2a}, \qquad f_{\min\,/\,\max}=-\frac{D}{4a}$$

BOTH ROOTS GREATER THAN K

$$D\ge 0, \qquad a\,f(k)>0, \qquad -\frac{b}{2a}>k$$

BOTH ROOTS LESS THAN K

$$D \geq 0, \qquad a\,f(k) > 0, \qquad -\frac{b}{2a} < k$$

K LIES BETWEEN THE ROOTS

$$a\,f(k) < 0$$

EXACTLY ONE ROOT IN (K_1, K_2)

$$f(k_1)\,f(k_2) < 0$$

BOTH ROOTS IN (K_1, K_2)

$$D \geq 0,\ \ af(k_1) > 0,\ \ af(k_2) > 0,\ \ k_1 < -\frac{b}{2a} < k_2$$

Sign of a quadratic

$$ax^2 + bx + c > 0\ \forall x \in \mathbb{R} \iff a > 0,\ D < 0; \qquad < 0\ \forall x \iff a < 0,\ D < 0$$

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CHAPTER 3

Sequence and Series

Arithmetic progression (AP)

NTH TERM

$$T_n = a + (n-1)d, \qquad T_n = S_n - S_{n-1}$$

SUM

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

THREE TERMS

$$a, b, c \text{ in AP} \iff 2b = a + c; \qquad \text{take } a-d,\ a,\ a+d$$

ARITHMETIC MEAN

$$A = \frac{a+b}{2}; \qquad n \text{ AMs between } a, b:\ d = \frac{b-a}{n+1}$$

Geometric progression (GP)

NTH TERM

$$T_n = ar^{n-1}$$

SUM

$$S_n = \frac{a(r^n - 1)}{r - 1}\ (r \neq 1), \qquad S_\infty = \frac{a}{1-r}\ (|r| < 1)$$

GEOMETRIC MEAN

$$a, b, c \text{ in GP} \iff b^2 = ac; \qquad G = \sqrt{ab}; \qquad n \text{ GMs: } r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

Harmonic progression and means

HP

$$a, b, c \text{ in HP} \iff \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ in AP} \iff b = \frac{2ac}{a+c}$$

AM ≥ GM ≥ HM

$$A \geq G \geq H\ (a, b > 0), \qquad G^2 = AH \text{ (two numbers)}$$

AM–GM FOR N NUMBERS

$$\frac{a_1 + a_2 + \cdots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \cdots a_n}, \text{ equality iff all equal}$$

Special series

NATURAL NUMBERS

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

AGP SUM TO INFINITY

$$a + (a+d)r + (a+2d)r^2 + \cdots = \frac{a}{1-r} + \frac{dr}{(1-r)^2} \quad (|r| < 1)$$

METHOD OF DIFFERENCES

$$T_k = f(k) - f(k-1) \;\Rightarrow\; \sum_{k=1}^{n} T_k = f(n) - f(0)$$

If the differences of successive terms form an AP or GP, write T_n as a polynomial or exponential in n first.

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CHAPTER 4

Complex Numbers

Basics, conjugate and modulus

FORM

$$z = x + iy, \qquad i^2 = -1, \qquad \bar{z} = x - iy, \qquad |z| = \sqrt{x^2 + y^2}$$

CONJUGATE

$$z\bar{z} = |z|^2, \qquad \overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2, \qquad \overline{z_1 z_2} = \bar{z}_1 \bar{z}_2, \qquad \operatorname{Re}(z) = \frac{z + \bar{z}}{2}$$

MODULUS

$$|z_1 z_2| = |z_1||z_2|, \qquad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \qquad |z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \bar{z}_2)$$

POWERS OF I

$$i^{4k} = 1,\ i^{4k+1} = i,\ i^{4k+2} = -1,\ i^{4k+3} = -i$$

Polar form and argument

POLAR / EULER FORM

$$z = r(\cos\theta + i\sin\theta) = re^{i\theta}, \qquad r = |z|,\ \theta = \arg z \in (-\pi, \pi]$$

ARGUMENT RULES

$$\arg(z_1 z_2) = \arg z_1 + \arg z_2, \qquad \arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2 \pmod{2\pi}, \qquad \arg\bar{z} = -\arg z$$

De Moivre's theorem and roots of unity

DE MOIVRE

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

CUBE ROOTS OF UNITY

$$1,\ \omega,\ \omega^2; \qquad \omega = \frac{-1 + i\sqrt{3}}{2}, \qquad 1 + \omega + \omega^2 = 0, \qquad \omega^3 = 1$$

NTH ROOTS OF UNITY

$$z_k = e^{\frac{2\pi i k}{n}},\ k = 0, 1, \ldots, n-1; \qquad \sum z_k = 0,\ \prod z_k = (-1)^{n-1}$$

Geometry and rotation

DISTANCE AND SECTION

$$|z_1 - z_2| = \text{distance}; \qquad z = \frac{mz_2 + nz_1}{m + n}$$

ROTATION

$$\frac{z_3 - z_1}{z_2 - z_1} = \frac{|z_3 - z_1|}{|z_2 - z_1|} e^{i\theta}$$

θ is the angle from $z_2 - z_1$ to $z_3 - z_1$ (anticlockwise).

CIRCLE

$$|z - z_0| = r; \qquad \left|\frac{z - z_1}{z - z_2}\right| = k \ (k \neq 1) \text{ is a circle, } k = 1 \text{ a line}$$

COLLINEAR POINTS

$$\frac{z_3 - z_1}{z_2 - z_1} \in \mathbb{R}$$

TRIANGLE INEQUALITY

$$\big||z_1| - |z_2|\big| \leq |z_1 + z_2| \leq |z_1| + |z_2|$$

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CHAPTER 5

Permutations and Combinations

Counting basics

PERMUTATIONS AND COMBINATIONS

$$ {}^{n}P_r = \frac{n!}{(n-r)!}, \qquad {}^{n}C_r = \frac{n!}{r!\,(n-r)!} $$

PROPERTIES

$$ {}^{n}C_r = {}^{n}C_{n-r}, \qquad {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r, \qquad r\,{}^{n}C_r = n\,{}^{n-1}C_{r-1} $$

WITH REPETITION

$$ \text{arrangements of } r \text{ from } n \text{ with repetition} = n^r $$

IDENTICAL OBJECTS

$$ \frac{n!}{p!\,q!\,r!\cdots} \text{ (p alike, q alike, } \ldots) $$

Circular permutations

$$ (n-1)! \text{ ways;} \qquad \frac{(n-1)!}{2} \text{ if clockwise and anticlockwise are the same (necklace)} $$

Selections and distribution

TOTAL SELECTIONS

$$ {}^{n}C_0 + {}^{n}C_1 + \cdots + {}^{n}C_n = 2^n; \qquad \text{at least one: } 2^n - 1 $$

ALIKE OBJECTS

$$ (p+1)(q+1)(r+1) - 1 \text{ selections from p, q, r alike objects (at least one)} $$

N IDENTICAL INTO R GROUPS

$$ {}^{n+r-1}C_{r-1} \text{ (empty allowed)}, \qquad {}^{n-1}C_{r-1} \text{ (none empty)} $$

NON-NEGATIVE INTEGER SOLUTIONS

$$ x_1 + x_2 + \cdots + x_r = n : \ {}^{n+r-1}C_{r-1} $$

DISTINCT INTO GROUPS

$$ \frac{(m+n)!}{m!\,n!} \ (m \neq n); \qquad \frac{(2n)!}{2!\,(n!)^2} \text{ (two equal groups)} $$

NUMBER OF DIVISORS

$$ N = p_1^{a_1} p_2^{a_2} \cdots : \ \ (a_1+1)(a_2+1)\cdots \text{ divisors} $$

Derangements

$$D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right)$$

$D_1 = 0,\ D_2 = 1,\ D_3 = 2,\ D_4 = 9,\ D_5 = 44.$

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CHAPTER 6

Binomial Theorem

Expansion and general term

BINOMIAL THEOREM

$$(x+a)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} a^r$$

GENERAL TERM

$$T_{r+1} = {}^nC_r \, x^{n-r} a^r$$

MIDDLE TERM

$$n \text{ even: } T_{\frac{n}{2}+1}; \qquad n \text{ odd: } T_{\frac{n+1}{2}},\ T_{\frac{n+3}{2}}$$

TERM INDEPENDENT OF X

$$\text{put the power of } x \text{ in } T_{r+1} \text{ equal to } 0$$

GREATEST COEFFICIENT

$$n \text{ even: } {}^nC_{n/2}; \qquad n \text{ odd: } {}^nC_{(n-1)/2} = {}^nC_{(n+1)/2}$$

Binomial coefficients

SUMS

$$\sum_{r=0}^{n} {}^nC_r = 2^n, \qquad C_0 + C_2 + \cdots = C_1 + C_3 + \cdots = 2^{n-1}$$

WEIGHTED SUM

$$\sum_{r=0}^{n} r \, {}^nC_r = n \, 2^{n-1}$$

SQUARES

$$\sum_{r=0}^{n} \left({}^nC_r\right)^2 = {}^{2n}C_n$$

VANDERMONDE

$$\sum_{r} {}^mC_r \, {}^nC_{k-r} = {}^{m+n}C_k$$

NUMBER OF TERMS

$$(x+a)^n : \ n+1; \qquad (x_1 + \cdots + x_r)^n : \ {}^{n+r-1}C_{r-1}$$

Any index

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots, \qquad |x| < 1,\ n \in \mathbb{Q}$$

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CHAPTER 8

Matrices and Determinants

Matrix basics

TRANSPOSE

$$(A^T)^T = A, \qquad (AB)^T = B^T A^T$$

SYMMETRIC / SKEW

$$A^T = A \text{ symmetric}; \qquad A^T = -A \text{ skew-symmetric}; \qquad A = \tfrac{1}{2}(A + A^T) + \tfrac{1}{2}(A - A^T)$$

TRACE

$$\operatorname{tr}(A + B) = \operatorname{tr}A + \operatorname{tr}B, \qquad \operatorname{tr}(AB) = \operatorname{tr}(BA)$$

SPECIAL MATRICES

$$A^2 = A \text{ idempotent}, \qquad A^k = O \text{ nilpotent}, \qquad A^2 = I \text{ involutory}, \qquad AA^T = I \text{ orthogonal}$$

Determinants

PROPERTIES

$$|A^T| = |A|, \qquad |AB| = |A||B|, \qquad |kA| = k^n |A| \ (A \text{ is } n \times n)$$

ROW OPERATIONS

swap two rows: sign changes; two equal rows: $|A| = 0$; $R_i \to R_i + kR_j$: no change

AREA OF A TRIANGLE

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Adjoint and inverse

INVERSE

$$A^{-1} = \frac{\operatorname{adj} A}{|A|}, \qquad |A| \neq 0$$

ADJOINT

$$A(\operatorname{adj}A) = (\operatorname{adj}A)A = |A|I, \qquad |\operatorname{adj}A| = |A|^{n-1}, \qquad \operatorname{adj}(\operatorname{adj}A) = |A|^{n-2}A$$

RULES

$$(AB)^{-1} = B^{-1}A^{-1}, \qquad |A^{-1}| = \frac{1}{|A|}, \qquad \operatorname{adj}(AB) = \operatorname{adj}B \operatorname{adj}A$$

Systems of linear equations

CRAMER'S RULE

$$x = \frac{\Delta_1}{\Delta},\ y = \frac{\Delta_2}{\Delta},\ z = \frac{\Delta_3}{\Delta}$$

CONSISTENCY

$$\Delta \neq 0 : \text{unique solution}; \qquad \Delta = 0, \text{ some } \Delta_i \neq 0 : \text{no solution};$$

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0 : \text{infinitely many or none}$$

HOMOGENEOUS

$$AX = O :\ \Delta \neq 0 \Rightarrow \text{only } X = O; \qquad \Delta = 0 \Rightarrow \text{non-trivial solutions}$$

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CHAPTER 9

Trigonometric Ratios and Identities

Basic identities

PYTHAGOREAN

$$\sin^2\theta + \cos^2\theta = 1, \qquad 1 + \tan^2\theta = \sec^2\theta, \qquad 1 + \cot^2\theta = \csc^2\theta$$

ALLIED ANGLES

$$\sin(\pi - \theta) = \sin\theta,\ \cos(\pi - \theta) = -\cos\theta,\ \sin\left(\tfrac{\pi}{2} - \theta\right) = \cos\theta$$

ASTC: all ratios positive in quadrant I, sin in II, tan in III, cos in IV.

Compound angles

SINE AND COSINE

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B, \qquad \cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

TANGENT

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

PRODUCTS

$$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B, \qquad \cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$$

Multiple and sub-multiple angles

DOUBLE ANGLE

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1 + \tan^2 A}, \qquad \cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}, \qquad \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

TRIPLE ANGLE

$$\sin 3A = 3\sin A - 4\sin^3 A, \qquad \cos 3A = 4\cos^3 A - 3\cos A, \qquad \tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

HALF ANGLE

$$\sin^2\frac{A}{2} = \frac{1 - \cos A}{2}, \qquad \cos^2\frac{A}{2} = \frac{1 + \cos A}{2}, \qquad \tan\frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

Sum–product formulas

SUM TO PRODUCT

$$\sin C + \sin D = 2\sin\frac{C + D}{2}\cos\frac{C - D}{2}, \qquad \cos C + \cos D = 2\cos\frac{C + D}{2}\cos\frac{C - D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}, \qquad \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

PRODUCT TO SUM

$$2\sin A\cos B = \sin(A+B) + \sin(A-B), \qquad 2\cos A\cos B = \cos(A+B) + \cos(A-B),$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Standard results

VALUES

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}, \qquad \tan 15^\circ = 2-\sqrt{3}, \qquad \sin 18^\circ = \frac{\sqrt{5}-1}{4}, \qquad \cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

PRODUCT OF COSINES

$$\cos A\cos 2A\cos 4A\cdots\cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A}$$

SINES IN AP

$$\sum_{k=0}^{n-1}\sin(\alpha+k\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha+\frac{(n-1)\beta}{2}\right)$$

Same for cosines with $\cos$ in place of the last $\sin$.

THREE-ANGLE PRODUCT

$$\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta) = \tfrac{1}{4}\sin 3\theta$$

Conditional identities (A + B + C = π)

$$\tan A + \tan B + \tan C = \tan A\tan B\tan C$$

$$\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$$

$$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

Maximum and minimum

$$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$$

$$a\sin^2\theta + b\cos^2\theta \in [\min(a,b),\ \max(a,b)]$$

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CHAPTER 10

Trigonometric Equations

General solutions

$$\sin\theta = 0 \Rightarrow \theta = n\pi; \qquad \cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}; \qquad \tan\theta = 0 \Rightarrow \theta = n\pi$$

$$\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$$

$$\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha; \qquad \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$$

$$\sin^2\theta = \sin^2\alpha,\ \cos^2\theta = \cos^2\alpha,\ \tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$$

In all of these, $n \in \mathbb{Z}$.

a cos θ + b sin θ = c

$$\text{divide by } \sqrt{a^2+b^2}:\ \cos(\theta - \phi) = \frac{c}{\sqrt{a^2+b^2}}, \qquad \tan\phi = \frac{b}{a}$$

Solvable only when $|c| \le \sqrt{a^2+b^2}$.

Principal values and inequalities

PRINCIPAL SOLUTION

$$\text{solutions in } [0, 2\pi)$$

INEQUALITIES

$$\text{solve on one period using the graph, then add the period}$$

Example: $\sin x > \frac{1}{2} \Rightarrow x \in \left(2n\pi + \frac{\pi}{6},\ 2n\pi + \frac{5\pi}{6}\right)$.

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CHAPTER 11

Inverse Trigonometric Functions

Domains and principal ranges

$$\sin^{-1}x:\ [-1,1]\to\left[-\frac{\pi}{2},\frac{\pi}{2}\right],\qquad \cos^{-1}x:\ [-1,1]\to[0,\pi]$$

$$\tan^{-1}x:\ \mathbb{R}\to\left(-\frac{\pi}{2},\frac{\pi}{2}\right),\qquad \cot^{-1}x:\ \mathbb{R}\to(0,\pi)$$

$$\sec^{-1}x:\ |x|\geq 1\to[0,\pi]\setminus\{\tfrac{\pi}{2}\},\qquad \csc^{-1}x:\ |x|\geq 1\to\left[-\tfrac{\pi}{2},\tfrac{\pi}{2}\right]\setminus\{0\}$$

Properties

COMPLEMENTARY

$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2},\qquad \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2},\qquad \sec^{-1}x+\csc^{-1}x=\frac{\pi}{2}$$

NEGATIVE ARGUMENTS

$$\sin^{-1}(-x)=-\sin^{-1}x,\qquad \tan^{-1}(-x)=-\tan^{-1}x,\qquad \cos^{-1}(-x)=\pi-\cos^{-1}x,$$

$$\cot^{-1}(-x)=\pi-\cot^{-1}x$$

RECIPROCAL

$$\csc^{-1}x=\sin^{-1}\frac{1}{x},\qquad \sec^{-1}x=\cos^{-1}\frac{1}{x},\qquad \cot^{-1}x=\tan^{-1}\frac{1}{x}\ (x>0)$$

COMPOSITIONS

$$\sin(\sin^{-1}x)=x\ (|x|\leq 1);\qquad \sin^{-1}(\sin x)=x \text{ only for } x\in\left[-\tfrac{\pi}{2},\tfrac{\pi}{2}\right]$$

Sum and difference

$$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\quad(xy<1)$$

If $xy>1$ and $x,y>0$, add π.

$$\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\quad(xy>-1)$$

DOUBLE

$$2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}=\sin^{-1}\frac{2x}{1+x^2}=\cos^{-1}\frac{1-x^2}{1+x^2}\quad(0\leq x<1)$$

For $-1<x<0$ the first two still hold; the $\cos^{-1}$ form becomes $-\cos^{-1}\frac{1-x^2}{1+x^2}$.

TELESCOPING

$$\tan^{-1}\frac{a_{k+1}-a_k}{1+a_k a_{k+1}} = \tan^{-1} a_{k+1} - \tan^{-1} a_k$$

Write the general term this way to sum series of $\tan^{-1}$ terms.

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CHAPTER 12

Solutions of Triangles

Sine, cosine and projection rules

SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

COSINE RULE

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

PROJECTION

$$a = b\cos C + c\cos B$$

NAPIER'S ANALOGY

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

Half angles and area

HALF ANGLES

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \qquad \cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}, \qquad \tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$s = \frac{a+b+c}{2}$.

AREA

$$\Delta = \frac{1}{2}bc\sin A = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R} = rs$$

Circles of a triangle

CIRCUMRADIUS

$$R = \frac{a}{2\sin A} = \frac{abc}{4\Delta}$$

INRADIUS

$$r = \frac{\Delta}{s} = (s-a)\tan\frac{A}{2} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

EXRADII

$$r_1 = \frac{\Delta}{s-a} = s\tan\frac{A}{2}, \qquad \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$$

DISTANCES

$$OI^2 = R^2 - 2Rr, \qquad \text{distance of orthocentre from } A = 2R\cos A$$

MEDIAN

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

m–n rule

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta = n\cot B - m\cot C$$

D divides BC in the ratio $m : n$; θ is the angle ADC, $\alpha = \angle BAD$, $\beta = \angle DAC$.

Heights and distances

$$\tan(\text{angle of elevation}) = \frac{\text{height}}{\text{horizontal distance}}$$

Draw the figure, mark right angles, and use the sine rule in non-right triangles.

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CHAPTER 13

Sets and Relations

Sets

UNION AND INTERSECTION

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

THREE SETS

$$n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$$

DE MORGAN

$$(A \cup B)' = A' \cap B', \qquad (A \cap B)' = A' \cup B'$$

DIFFERENCE

$$A - B = A \cap B', \qquad n(A - B) = n(A) - n(A \cap B)$$

POWER SET

$$n(A) = m \;\Rightarrow\; n(P(A)) = 2^m$$

Relations

NUMBER OF RELATIONS

$$n(A) = m,\ n(B) = n: \quad 2^{mn} \text{ relations from } A \text{ to } B$$

TYPES

$$\text{reflexive: } (a,a) \in R\ \forall a; \qquad \text{symmetric: } (a,b) \in R \Rightarrow (b,a) \in R;$$

$$\text{transitive: } (a,b),(b,c) \in R \Rightarrow (a,c) \in R$$

COUNTS ON A SET OF N ELEMENTS

$$\text{reflexive: } 2^{n^2-n}, \qquad \text{symmetric: } 2^{\frac{n(n+1)}{2}}$$

EQUIVALENCE RELATION

$$\text{reflexive} + \text{symmetric} + \text{transitive}$$

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CHAPTER 14

Functions

Types of functions

COUNTING

$$|A| = m,\ |B| = n: \quad n^m \text{ functions}; \qquad {}^nP_m \text{ one-one } (n \geq m)$$

ONTO FUNCTIONS

$$\sum_{k=0}^{n} (-1)^k\, {}^nC_k\, (n-k)^m \quad \text{onto maps from } m \text{ to } n \text{ elements}$$

BIJECTION

$$\text{one-one and onto}; \qquad n! \text{ bijections when } |A| = |B| = n$$

Special functions

GREATEST INTEGER

$$[x] \leq x < [x] + 1, \qquad [x+n] = [x] + n\ (n \in \mathbb{Z}), \qquad [x] + [-x] = \begin{cases} 0, & x \in \mathbb{Z} \\ -1, & x \notin \mathbb{Z} \end{cases}$$

FRACTIONAL PART

$$\{x\} = x - [x] \in [0, 1)$$

SIGNUM

$$\operatorname{sgn} x = \frac{|x|}{x}\ (x \neq 0), \qquad \operatorname{sgn} 0 = 0$$

Even, odd and periodic

$$f(-x) = f(x) \text{ even}; \qquad f(-x) = -f(x) \text{ odd}; \qquad f(x) = \underbrace{\tfrac{f(x)+f(-x)}{2}}_{\text{even}} + \underbrace{\tfrac{f(x)-f(-x)}{2}}_{\text{odd}}$$

PERIODS

$$\sin x, \cos x:\ 2\pi; \qquad \tan x:\ \pi; \qquad |\sin x|:\ \pi; \qquad \{x\}:\ 1; \qquad f(ax+b):\ \frac{T}{|a|}$$

Composite and inverse

$$(f \circ g)(x) = f(g(x)), \qquad (f \circ g)^{-1} = g^{-1} \circ f^{-1}, \qquad f(f^{-1}(x)) = x$$

f^{-1} exists only if f is a bijection; its graph is the mirror image of f in $y = x$.

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CHAPTER 15

Limits

Standard limits

$$\lim_{x\to 0}\frac{\sin x}{x}=1, \qquad \lim_{x\to 0}\frac{\tan x}{x}=1, \qquad \lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1, \qquad \lim_{x\to 0}\frac{a^x-1}{x}=\ln a, \qquad \lim_{x\to 0}\frac{\ln(1+x)}{x}=1$$

$$\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}, \qquad \lim_{x\to 0}\frac{(1+x)^n-1}{x}=n$$

1^∞ form

$$\lim_{x\to a} f(x)=1,\ \lim g(x)=\infty \ \Rightarrow\ \lim f(x)^{g(x)}=e^{\lim g(x)\,(f(x)-1)}$$

$$\lim_{x\to 0}(1+x)^{1/x}=e, \qquad \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x=e^a$$

Expansions and L'Hospital's rule

SERIES

$$e^x=1+x+\frac{x^2}{2!}+\cdots, \qquad \ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$$

$$\sin x=x-\frac{x^3}{3!}+\cdots, \qquad \cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots, \qquad \tan x=x+\frac{x^3}{3}+\cdots$$

L'HOSPITAL

$$\frac{0}{0}\ \text{or}\ \frac{\infty}{\infty}:\ \lim\frac{f(x)}{g(x)}=\lim\frac{f'(x)}{g'(x)}$$

RATIONAL FUNCTION AT ∞

$$\lim_{x\to\infty}\frac{a_0x^m+\cdots}{b_0x^n+\cdots}=\begin{cases}0, & m<n\\ a_0/b_0, & m=n\\ \pm\infty, & m>n\end{cases}$$

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CHAPTER 16

Continuity and Differentiability

Continuity

$$f \text{ continuous at } a \iff \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

sums, products, quotients (denominator $\neq 0$) and compositions of continuous functions are continuous

INTERMEDIATE VALUE THEOREM

$$f \text{ continuous on } [a,b],\ f(a)f(b) < 0 \Rightarrow f(c) = 0 \text{ for some } c \in (a,b)$$

Differentiability

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \qquad \text{LHD=RHD}$$

differentiable$\Rightarrow$continuous; not conversely (e.g. $|x|$ at 0)

COMMON CHECKS

$|f(x)|$ may fail where $f(x) = 0$; $\qquad [x]$ discontinuous at integers

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CHAPTER 17

Differentiation

Standard derivatives

$$\frac{d}{dx}x^n = nx^{n-1}, \qquad \frac{d}{dx}e^x = e^x, \qquad \frac{d}{dx}a^x = a^x \ln a, \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$(\sin x)' = \cos x,\ (\cos x)' = -\sin x,\ (\tan x)' = \sec^2 x,\ (\sec x)' = \sec x \tan x,\ (\csc x)' = -\csc x \cot x,\ (\cot x)' = -\csc^2 x$$

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}, \qquad (\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}, \qquad (\tan^{-1} x)' = \frac{1}{1+x^2},$$
$$(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

Rules

$$(uv)' = u'v + uv', \qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}, \qquad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

PARAMETRIC

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \qquad \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\Big/\frac{dx}{dt}$$

LOGARITHMIC

$$y = u^v \ \Rightarrow\ \frac{y'}{y} = v'\ln u + \frac{v\,u'}{u}$$

IMPLICIT

$$F(x,y) = 0 \ \Rightarrow\ \frac{dy}{dx} = -\frac{F_x}{F_y}$$

Higher derivatives

LEIBNIZ

$$(uv)^{(n)} = \sum_{k=0}^{n} {}^nC_k\, u^{(k)} v^{(n-k)}$$

$$\frac{d^n}{dx^n}\sin(ax+b) = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$$

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CHAPTER 18

Application of Derivatives

Tangent and normal

TANGENT

$$y - y_1 = m(x - x_1), \qquad m = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$$

NORMAL

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

ANGLE BETWEEN CURVES

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|; \qquad \text{orthogonal if } m_1 m_2 = -1$$

Rate of change and approximation

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}, \qquad \Delta y \approx f'(x)\,\Delta x$$

Monotonicity

$$f'(x) > 0 \text{ on } I \Rightarrow f \text{ increasing}; \qquad f'(x) < 0 \Rightarrow \text{decreasing}$$

Maxima and minima

FIRST DERIVATIVE TEST

$$f' \text{ changes } + \to - : \text{local max}; \qquad - \to + : \text{local min}$$

SECOND DERIVATIVE TEST

$$f'(c) = 0,\ f''(c) < 0 : \max; \qquad f''(c) > 0 : \min$$

CLOSED INTERVAL

$$\text{absolute extrema on } [a, b] : \text{compare } f \text{ at critical points and at } a, b$$

Mean value theorems

ROLLE'S THEOREM

$$f \text{ continuous on } [a, b], \text{differentiable on } (a, b),\ f(a) = f(b) \Rightarrow f'(c) = 0$$

LAGRANGE'S MVT

$$f'(c) = \frac{f(b) - f(a)}{b - a}, \qquad c \in (a, b)$$

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CHAPTER 19

Indefinite Integration

Standard integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1), \qquad \int \frac{dx}{x} = \ln|x| + C, \qquad \int e^x dx = e^x + C,$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \tan x\,dx = \ln|\sec x| + C, \qquad \int \sec x\,dx = \ln|\sec x + \tan x| + C,$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + C$$

Special integrals

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C, \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C, \qquad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx) + C$$

Methods

BY PARTS

$$\int u\,v\,dx = u\int v\,dx - \int\left(u'\int v\,dx\right)dx$$

Choose u in the order ILATE: inverse trig, log, algebraic, trig, exponential.

$$\int e^x\big(f(x) + f'(x)\big)dx = e^x f(x) + C$$

SUBSTITUTION

$$\int f(g(x))\,g'(x)\,dx = \int f(t)\,dt, \qquad t = g(x)$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C, \qquad \int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + C$$

RATIONAL FUNCTIONS

$$\frac{P(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

Use partial fractions after making the numerator's degree less than the denominator's.

TRIG FORM

$$\int \frac{dx}{a + b\cos x} : \quad t = \tan\frac{x}{2}, \quad \cos x = \frac{1-t^2}{1+t^2}, \; dx = \frac{2\,dt}{1+t^2}$$

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CHAPTER 20

Definite Integration

Properties

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx, \qquad \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & f \text{ even} \\ 0, & f \text{ odd} \end{cases}$$

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & f(2a-x) = f(x) \\ 0, & f(2a-x) = -f(x) \end{cases}$$

PERIODIC

$$\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$$

KING'S RULE EXAMPLE

$$\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}dx = \frac{\pi}{4}$$

Newton–Leibniz rule

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\,h'(x) - f(g(x))\,g'(x)$$

Limit as a sum

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)\,dx$$

Standard results

WALLIS

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\times\begin{cases}\frac{\pi}{2}, & n \text{ even} \\ 1, & n \text{ odd}\end{cases}$$

$$\int_0^{\pi/2} \ln\sin x\,dx = -\frac{\pi}{2}\ln 2$$

SPECIAL FUNCTIONS

$$\int_a^b [x]\,dx : \text{split at integers;} \qquad \int |f(x)|\,dx : \text{split where } f = 0$$

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CHAPTER 21

Area Under Curves

Area bounded by curves

UNDER A CURVE

$$A = \int_a^b |f(x)|\,dx \qquad \text{or} \qquad A = \int_c^d |g(y)|\,dy$$

BETWEEN TWO CURVES

$$A = \int_a^b \big(f(x) - g(x)\big)\,dx, \qquad f(x) \geq g(x) \text{ on } [a, b]$$

STANDARD AREAS

$$\text{ellipse } \pi ab; \qquad y^2 = 4ax \text{ and } x^2 = 4by : \ \frac{16ab}{3}$$

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CHAPTER 22

Differential Equations

Order, degree and formation

order=highest derivative; degree=its power, once the equation is polynomial in derivatives

FORMATION

$$n \text{ arbitrary constants} \Rightarrow \text{differentiate } n \text{ times and eliminate them (order } n)$$

Variable separable and homogeneous

SEPARABLE

$$f(y)\,dy = g(x)\,dx \;\Rightarrow\; \int f(y)\,dy = \int g(x)\,dx + C$$

HOMOGENEOUS

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right): \quad y = vx, \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Linear differential equations

$$\frac{dy}{dx} + P(x)\,y = Q(x): \quad \text{IF} = e^{\int P\,dx}, \qquad y\cdot\text{IF} = \int Q\cdot\text{IF}\,dx + C$$

BERNOULLI

$$\frac{dy}{dx} + Py = Qy^n: \text{ put } z = y^{1-n}$$

EXACT FORMS

$$x\,dy + y\,dx = d(xy), \qquad \frac{x\,dy - y\,dx}{x^2} = d\left(\frac{y}{x}\right), \qquad \frac{x\,dy - y\,dx}{x^2 + y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$$

Applications

GROWTH AND DECAY

$$\frac{dN}{dt} = kN \;\Rightarrow\; N = N_0 e^{kt}$$

NEWTON'S LAW OF COOLING

$$\frac{dT}{dt} = -k(T - T_s)$$

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CHAPTER 23

Straight Lines

Distance, section and locus

DISTANCE

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

SECTION FORMULA

$$\left(\frac{mx_2 + nx_1}{m+n},\ \frac{my_2 + ny_1}{m+n}\right) \text{ internal;} \qquad m \to -m \text{ for external}$$

AREA OF A TRIANGLE

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Centres of a triangle

CENTROID

$$G = \left(\frac{x_1 + x_2 + x_3}{3},\ \frac{y_1 + y_2 + y_3}{3}\right)$$

INCENTRE

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c},\ \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

EULER LINE

$$O,\ G,\ H \text{ collinear;} \qquad HG : GO = 2 : 1$$

O circumcentre, H orthocentre.

Equations of a line

$$y = mx + c; \qquad y - y_1 = m(x - x_1); \qquad \frac{x}{a} + \frac{y}{b} = 1; \qquad x\cos\alpha + y\sin\alpha = p$$

TWO POINTS

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

PARAMETRIC

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

Angle, distance and position

ANGLE BETWEEN LINES

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|; \qquad \parallel:\ m_1 = m_2; \qquad \perp:\ m_1 m_2 = -1$$

POINT TO LINE

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

PARALLEL LINES

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

FOOT AND IMAGE

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2} \text{ (foot)}, \qquad = -\frac{2(ax_1 + by_1 + c)}{a^2 + b^2} \text{ (image)}$$

SAME SIDE

$$P, Q \text{ on the same side of } ax + by + c = 0 \iff (ax_1 + by_1 + c)(ax_2 + by_2 + c) > 0$$

Family of lines and pair of lines

FAMILY

$$L_1 + \lambda L_2 = 0 \text{ passes through } L_1 \cap L_2$$

CONCURRENCY

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

ANGLE BISECTORS

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

PAIR OF LINES

$$ax^2 + 2hxy + by^2 = 0: \quad \tan\theta = \frac{2\sqrt{h^2 - ab}}{|a + b|}; \qquad \perp \text{ if } a + b = 0$$

REPRESENTS A PAIR

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0: \quad abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

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CHAPTER 24

Circle

Equations

CENTRE–RADIUS

$$(x-h)^2+(y-k)^2=r^2$$

GENERAL

$$x^2+y^2+2gx+2fy+c=0:\ \text{centre}\ (-g,-f),\ r=\sqrt{g^2+f^2-c}$$

DIAMETER FORM

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

PARAMETRIC

$$x=h+r\cos\theta,\quad y=k+r\sin\theta$$

Point, line and power

POWER OF A POINT

$$S_1=x_1^2+y_1^2+2gx_1+2fy_1+c:\quad S_1<0\ \text{inside},\ =0\ \text{on},\ >0\ \text{outside}$$

LINE AND CIRCLE

$$\text{perpendicular distance from centre}\ <,=,>\ r:\ \text{secant, tangent, misses}$$

TANGENT CONDITION

$$y=mx+c\ \text{touches}\ x^2+y^2=a^2\iff c^2=a^2(1+m^2)$$

Tangent, normal and chord

TANGENT AT A POINT

$$T=0:\quad xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

LENGTH OF TANGENT

$$\sqrt{S_1}$$

CHORD WITH GIVEN MIDPOINT

$$T=S_1$$

CHORD OF CONTACT

$$T=0\ (\text{from an outside point})$$

PAIR OF TANGENTS

$$SS_1=T^2$$

NORMAL

always passes through the centre

Two circles

COMMON TANGENTS

$$d > r_1 + r_2 : 4;\ \ d = r_1 + r_2 : 3;\ \ |r_1 - r_2| < d < r_1 + r_2 : 2;\ \ d = |r_1 - r_2| : 1;\ \ d < |r_1 - r_2| : 0$$

ORTHOGONAL CIRCLES

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

RADICAL AXIS / COMMON CHORD

$$S_1 - S_2 = 0$$

FAMILY

$$S + \lambda L = 0, \qquad S_1 + \lambda S_2 = 0$$

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CHAPTER 25

Parabola

Standard parabola y² = 4ax

$$\text{focus } (a, 0),\ \text{directrix } x = -a,\ \text{latus rectum } 4a,\ \text{vertex } (0, 0)$$

PARAMETRIC

$$(at^2,\ 2at)$$

FOCAL DISTANCE

$$SP = x + a$$

FOCAL CHORD

$$t_1 t_2 = -1; \qquad \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$$

Tangent, normal and chord

TANGENT

$$yy_1 = 2a(x + x_1); \qquad ty = x + at^2; \qquad y = mx + \frac{a}{m}$$

NORMAL

$$y = -tx + 2at + at^3; \qquad y = mx - 2am - am^3$$

CHORD THROUGH T_1, T_2

$$2x - (t_1 + t_2)y + 2at_1 t_2 = 0$$

NORMAL MEETS AGAIN

$$t_2 = -t_1 - \frac{2}{t_1}$$

TANGENTS' INTERSECTION

$$\left(at_1 t_2,\ a(t_1 + t_2)\right)$$

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CHAPTER 26

Ellipse

Standard ellipse x²/a² + y²/b² = 1 (a > b)

$$b^2 = a^2(1-e^2), \qquad \text{foci } (\pm ae, 0), \qquad \text{directrices } x = \pm\frac{a}{e}, \qquad \text{latus rectum } \frac{2b^2}{a}$$

PARAMETRIC

$$(a\cos\theta,\ b\sin\theta)$$

FOCAL DISTANCES

$$SP = a - ex, \quad S'P = a + ex, \quad SP + S'P = 2a$$

Tangent, normal and chord

TANGENT

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1; \qquad \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1; \qquad y = mx \pm \sqrt{a^2m^2 + b^2}$$

NORMAL

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2; \qquad ax\sec\theta - by\csc\theta = a^2 - b^2$$

DIRECTOR CIRCLE

$$x^2 + y^2 = a^2 + b^2$$

CHORD WITH MIDPOINT

$$T = S_1$$

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CHAPTER 27

Hyperbola

Standard hyperbola x²/a² − y²/b² = 1

$$b^2 = a^2(e^2 - 1), \qquad \text{foci } (\pm ae, 0), \qquad \text{latus rectum } \frac{2b^2}{a}, \qquad |SP - S'P| = 2a$$

PARAMETRIC

$$(a\sec\theta,\ b\tan\theta)$$

ASYMPTOTES

$$y = \pm\frac{b}{a}x$$

RECTANGULAR HYPERBOLA

$$a = b,\ e = \sqrt{2}; \qquad xy = c^2 :\ (ct,\ \tfrac{c}{t})$$

CONJUGATE

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1; \qquad \frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

Tangent, normal and chord

TANGENT

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1; \qquad y = mx \pm \sqrt{a^2m^2 - b^2}$$

NORMAL

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$$

DIRECTOR CIRCLE

$$x^2 + y^2 = a^2 - b^2$$

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CHAPTER 28

Vector Algebra

Vectors, addition and section formula

MAGNITUDE AND UNIT VECTOR

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \qquad |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}, \qquad \hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

SECTION FORMULA

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n} \text{ (internal)}$$

COLLINEAR

$$\vec{a} = \lambda\vec{b}; \qquad \text{points collinear} \iff \overrightarrow{AB} = \lambda\overrightarrow{AC}$$

Scalar (dot) product

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$$

PROJECTION

$$\text{projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0; \qquad |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b}$$

Vector (cross) product

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

AREA

$$\text{parallelogram } |\vec{a}\times\vec{b}|; \qquad \text{triangle } \tfrac{1}{2}|\vec{a}\times\vec{b}|$$

LAGRANGE'S IDENTITY

$$|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$$

Triple products

SCALAR TRIPLE PRODUCT

$$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Volume of the parallelepiped; zero iff the vectors are coplanar. Tetrahedron volume is $\frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}]$.

VECTOR TRIPLE PRODUCT

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$$

$$[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$$

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CHAPTER 29

Three Dimensional Geometry

Direction cosines and ratios

$$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma; \qquad l^2 + m^2 + n^2 = 1$$

FROM DIRECTION RATIOS

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}},\ \dots$$

ANGLE BETWEEN LINES

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Lines in space

EQUATION

$$\vec{r} = \vec{a} + \lambda\vec{b}; \qquad \frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

SHORTEST DISTANCE (SKEW LINES)

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}$$

PARALLEL LINES

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

COPLANAR LINES

$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$$

Planes

EQUATIONS

$$ax + by + cz + d = 0; \qquad \vec{r}\cdot\hat{n} = p; \qquad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

THROUGH THREE POINTS

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

POINT TO PLANE

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

PARALLEL PLANES

$$d = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

FAMILY

$P_1 + \lambda P_2 = 0$ passes through the line $P_1 = P_2 = 0$

Line and plane

ANGLE

$$\sin\theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$$

FOOT AND IMAGE OF A POINT

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = -\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2} \text{ (foot)}; \qquad \times 2 \text{ for the image}$$

LINE LIES IN PLANE

$\vec{b} \cdot \vec{n} = 0$ and a point of the line satisfies the plane

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CHAPTER 30

Probability

Classical probability

$$P(A) = \frac{n(A)}{n(S)}, \qquad P(A') = 1 - P(A)$$

ADDITION

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

THREE EVENTS

$$P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

Conditional probability and independence

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(A \cap B) = P(A)\,P(B \mid A)$$

INDEPENDENT EVENTS

$$P(A \cap B) = P(A)\,P(B)$$

Total probability and Bayes' theorem

TOTAL PROBABILITY

$$P(A) = \sum_i P(E_i)\,P(A \mid E_i)$$

BAYES

$$P(E_i \mid A) = \frac{P(E_i)\,P(A \mid E_i)}{\sum_j P(E_j)\,P(A \mid E_j)}$$

Random variables and binomial distribution

MEAN AND VARIANCE

$$\mu = \sum x_i p_i, \qquad \sigma^2 = \sum x_i^2 p_i - \mu^2$$

BINOMIAL

$$P(X = r) = {}^{n}C_r\, p^r q^{n-r}, \qquad q = 1 - p; \qquad \mu = np, \;\; \sigma^2 = npq$$

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CHAPTER 31

Statistics

Mean, median and mode

MEAN

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

MEDIAN (GROUPED)

$$M = l + \frac{\frac{N}{2} - C}{f} \times h$$

EMPIRICAL RELATION

$$\text{Mode} \approx 3\,\text{Median} - 2\,\text{Mean}$$

Mean deviation

$$\text{MD about } a = \frac{\sum f_i |x_i - a|}{\sum f_i}$$

Mean deviation is least about the median.

Variance and standard deviation

$$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N} = \frac{\sum f_i x_i^2}{N} - \bar{x}^2$$

CHANGE OF ORIGIN AND SCALE

$$y = ax + b: \quad \bar{y} = a\bar{x} + b, \qquad \sigma_y = |a|\,\sigma_x$$

FIRST N NATURAL NUMBERS

$$\text{variance} = \frac{n^2 - 1}{12}$$

COMBINED GROUPS

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}, \qquad d_i = \bar{x}_i - \bar{x}$$

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CHAPTER 32

Mathematical Reasoning

Statements and connectives

$$p \to q \equiv \sim p \vee q; \qquad \sim (p \wedge q) \equiv \sim p \vee \sim q; \qquad \sim (p \vee q) \equiv \sim p \wedge \sim q$$

CONTRAPOSITIVE

$$p \to q \equiv \sim q \to \sim p$$

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$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p); \qquad \sim (p \to q) \equiv p \wedge \sim q$$

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CHAPTER 33

Mathematical Induction

Principle of mathematical induction

$$P(1) \text{ true and } \big(P(k) \Rightarrow P(k+1)\big) \;\Rightarrow\; P(n) \text{ true } \forall n \in \mathbb{N}$$

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CHAPTER 34

Linear Programming

Linear programming problems

optimise $Z = ax + by$ subject to linear constraints; the optimum is at a corner of the feasible region

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