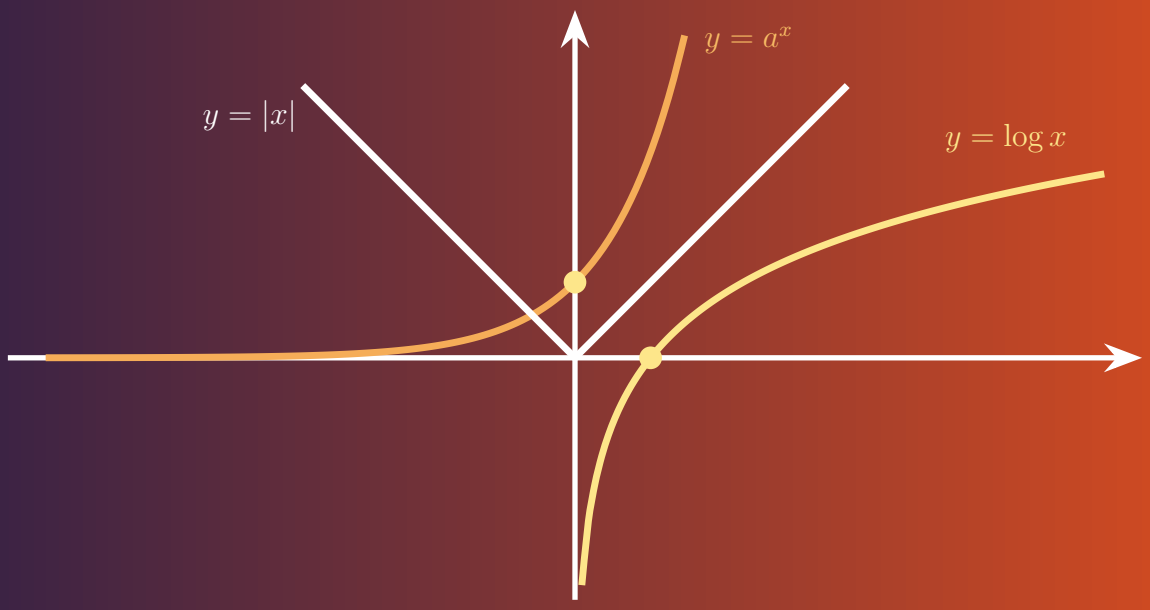
PadhoNa

Padho samajh ke. Yaad karo khud se.

MATHEMATICS • CLASS 11 • CHAPTER 1

Basic Mathematics

Complete notes from basics to advanced
for Class 11, JEE Main and JEE Advanced



Simple Hinglish explanations • Number lines and graphs • Fully solved questions
Active Recall quizzes • Master formula sheet

How to use these notes

Padhna aur **yaad karna** do alag cheezein hain. Sirf page palatne se lagta hai sab aa gaya, par exam mein dimaag blank ho jata hai. Isliye har topic ko is order mein karo:

1. **Concept padho**, phir uska **Hinglish** box padho taaki baat dimaag mein baith jaye.
2. **Solved question** ko pehle khud try karo. Solution tabhi dekho jab do minute try kar liya ho.
3. **Active Recall** box aate hi notes band karo aur copy mein likho. Answers last mein milenge.
4. Hafte ke end mein **Master Formula Sheet** se revise karo aur **PadhoNa Test Series** se apna level check karo.

Concept

Blue box: definition aur main theory.

Hinglish mein samjho

Yellow box: wahi baat simple Hinglish mein.

Yaad rakho: Formula

Green box: formula jo yaad rakhna hai.

Galti mat karna

Red box: common galtiyan.

PadhoNa Shortcut

Pink box: shortcut aur fast method.

Purple box: Active Recall. Yahan rukna compulsory hai!

Note: All variables are real unless told otherwise. In this chapter, $\log$ with no base shown means base 10 when a question gives a base-10 value; elsewhere any one valid base may be used. On number lines, a hollow dot means the point is *not* included, and a filled dot means it is included.

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1. Algebraic Identities

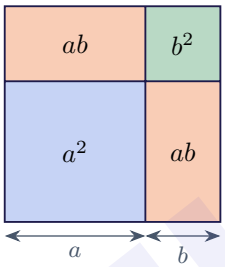
Yeh chapter kyun zaroori hai?

Basic Maths naam se basic lagta hai, par poore JEE Maths aur Physics ki neev yahi hai. Identities, logarithm, inequalities aur modulus aage har chapter mein baar-baar aayenge. Ek baar yeh pakka ho gaya, to aage ke chapters bahut aasan lagenge.

1.1 Squares

Identities I

$(a+b)^2 = a^2 + b^2 + 2ab$
$(a-b)^2 = a^2 + b^2 - 2ab$
$a^2 - b^2 = (a-b)(a+b)$
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$



Side $(a+b)$ ke square ka area:
$(a+b)^2 = a^2 + 2ab + b^2$

Q.1 • Using the square identity

If $a+b=8$ and $ab=5$, find (1) a^2+b^2; (2) a^4+b^4.

Solution. (1) Square both sides of $a+b=8$:

$$a^2+b^2+2ab = 64 \;\Rightarrow\; a^2+b^2+2(5) = 64 \;\Rightarrow\; a^2+b^2 = 54$$

(2) Square again: $(a^2+b^2)^2 = 54^2$, so $a^4+b^4+2a^2b^2 = 2916$.

$$a^4+b^4 = 2916 - 2(5)^2 = 2916 - 50 = 2866$$

Answer: $54;\ 2866$

1.2 Cubes

Identities II

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
$a^3 + b^3 = (a+b)^3 - 3ab(a+b) = (a+b)(a^2+b^2-ab)$
$a^3 - b^3 = (a-b)^3 + 3ab(a-b) = (a-b)(a^2+b^2+ab)$

Proof of the factor form:

$$a^3+b^3 = (a+b)^3 - 3ab(a+b) = (a+b)\big[(a+b)^2 - 3ab\big] = (a+b)(a^2+b^2+2ab-3ab) = (a+b)(a^2+b^2-ab)$$

Q. 2 • Two methods

If $a+b=9$ and $a^2+b^2=41$, find a^3+b^3.

Solution. Squaring $a+b=9$: $41+2ab=81 \Rightarrow ab=20$.
Method 1: $a^3+b^3=(a+b)^3-3ab(a+b)=729-3(20)(9)=729-540=189$.
Method 2: $a^3+b^3=(a+b)(a^2+b^2-ab)=9(41-20)=9\times 21=189$.

Answer: 189

Q. 3 • Cube-root trick

If $x = 2 + 2^{1/3} + 2^{2/3}$, the value of $x^3 - 6x^2 + 6x + 1$ is
(a) 0 (b) 1 (c) 2 (d) 3

Solution. Move the integer to the left: $x - 2 = 2^{1/3} + 2^{2/3}$. Cube both sides and use $(p+q)^3 = p^3 + q^3 + 3pq(p+q)$:

$$(x-2)^3 = 2 + 4 + 3\cdot 2^{1/3}\cdot 2^{2/3}(x-2) = 6 + 6(x-2)$$

$$x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12 \;\Rightarrow\; x^3 - 6x^2 + 6x - 2 = 0$$

So $x^3 - 6x^2 + 6x + 1 = (x^3 - 6x^2 + 6x - 2) + 3 = 3$.

Answer: (d) 3

Hinglish mein samjho

$x =$ "number + cube roots" wale sawal mein pehle number ko left le jao, phir cube karo. Seedha x^3 nikalne ki koshish mat karna, bahut lamba ho jayega.

1.3 The $a^3 + b^3 + c^3$ identity

Identities III

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Q. 4 • Simplify

Simplify $\dfrac{(a^3 - b^3)^3 + (b^3 - c^3)^3 + (c^3 - a^3)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$.

Solution. **Denominator:** let $\alpha = a - b, \beta = b - c, \gamma = c - a$. Then $\alpha + \beta + \gamma = 0$, so the denominator equals $3(a - b)(b - c)(c - a)$.

Numerator: let $p = a^3 - b^3, q = b^3 - c^3, r = c^3 - a^3$. Again $p + q + r = 0$, so it equals $3(a^3 - b^3)(b^3 - c^3)(c^3 - a^3)$.

$$\frac{3(a - b)(a^2 + ab + b^2)(b - c)(b^2 + bc + c^2)(c - a)(c^2 + ca + a^2)}{3(a - b)(b - c)(c - a)}$$

Answer: $(a^2 + ab + b^2)(b^2 + bc + c^2)(c^2 + ca + a^2)$

Q. 5 • Symmetric values

If $a + b + c = 9$, $ab + bc + ca = 7$ and $abc = -3$, find (1) $a^2 + b^2 + c^2$; (2) $a^3 + b^3 + c^3$.

Solution. (1) $(a + b + c)^2 = 81 \Rightarrow a^2 + b^2 + c^2 + 2(7) = 81 \Rightarrow a^2 + b^2 + c^2 = 67$.

(2) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 9(67 - 7) = 540$.

$$a^3 + b^3 + c^3 = 540 + 3(-3) = 531$$

Answer: 67; 531

Q. 6 • Brain Teaser

Let a, b, c be non-zero reals with $\dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{a} = 5$ and $\dfrac{b}{a} + \dfrac{c}{b} + \dfrac{a}{c} = 6$. Then $\dfrac{a^3}{b^3} + \dfrac{b^3}{c^3} + \dfrac{c^3}{a^3} - 3$ equals

(a) 25 (b) 30 (c) 35 (d) 40

Solution. Let $p = \dfrac{a}{b}, q = \dfrac{b}{c}, r = \dfrac{c}{a}$. Then $p + q + r = 5$ and $pqr = 1$.

Also $\dfrac{1}{p} + \dfrac{1}{q} + \dfrac{1}{r} = 6 \Rightarrow \dfrac{pq + qr + rp}{pqr} = 6 \Rightarrow pq + qr + rp = 6$.

Squaring the first: $p^2 + q^2 + r^2 + 2(6) = 25 \Rightarrow p^2 + q^2 + r^2 = 13$.

$$p^3 + q^3 + r^3 - 3pqr = (p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp) = 5(13 - 6) = 35$$

Since $pqr = 1$, the required expression $p^3 + q^3 + r^3 - 3 = 35$.

Answer: (c) 35

PadhoNa Active Recall #1 **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Write the identity for $a^3 + b^3$ in factor form.
2. If $a + b = 6$ and $ab = 4$, find $a^2 + b^2$.
3. If $a + b + c = 0$, what is $a^3 + b^3 + c^3$?
4. Find $a^3 - b^3$ if $a - b = 2$, $ab = 3$.
5. Expand $(a + b + c)^2$.

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

2. Exponents (Powers or Indices)

Hinglish mein samjho

a^x mein a base hai aur x power. Yeh rules itne use honge ki neend mein bhi yaad hone chahiye. Same base ho to powers add ya subtract hoti hain, bas itna samajh lo.

Laws of exponents

#	Law	Example
1	$a^x \cdot a^y = a^{x+y}$	$2^5 \cdot 2^9 = 2^{14}$
2	$\dfrac{a^x}{a^y} = a^{x-y}$	$\dfrac{7^{12}}{7^4} = 7^8$
3	$a^0 = 1\ (a \neq 0)$	$9^0 = 1$
4	$(a^x)^y = a^{xy}$	$(2^4)^6 = 2^{24}$
5	$a^{-x} = \left(\dfrac{1}{a}\right)^x = \dfrac{1}{a^x}$	$\dfrac{1}{2^9} = 2^{-9}$
6	$\left(\dfrac{a}{b}\right)^x = \left(\dfrac{b}{a}\right)^{-x}$	$\left(\dfrac{2}{7}\right)^5 = \left(\dfrac{7}{2}\right)^{-5}$
7	$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$	$\left(\dfrac{2}{7}\right)^5 = \dfrac{2^5}{7^5}$
8	$\sqrt{a} = a^{1/2}$	$\sqrt{7} = 7^{1/2}$
9	$\sqrt{ab} = a^{1/2}b^{1/2}\ \ (a, b \geq 0)$	$\sqrt{6} = 2^{1/2}\,3^{1/2}$
10	$(ab)^x = a^x b^x$	$6^3 = 2^3 \cdot 3^3$
11	$\sqrt[3]{ab} = (ab)^{1/3} = a^{1/3}b^{1/3}$	$\sqrt[3]{10} = 2^{1/3}\,5^{1/3}$

Taking common: $2^1 + 2^2 + 2^3 = 2^1[1 + 2 + 2^2]$, $\quad 5^7 + 5^{15} = 5^7[1 + 5^8]$.

Q. 7 • Nested powers

If $\left[\left\{\left(\frac{1}{5^2}\right)^{-3}\right\}^{-1/2}\right]^{1/3} = 5^x$, find the value of $2 - 3x$.

Solution. $\frac{1}{5^2} = 5^{-2}$. Multiply the powers: $(-2)(-3)\left(-\frac{1}{2}\right)\left(\frac{1}{3}\right) = -1$.

So $5^{-1} = 5^x \Rightarrow x = -1$, and $2 - 3x = 2 + 3 = 5$.

Answer: 5

Q. 8 • Two brackets

If $\left(\frac{x^4y^{-2}}{x^{-2}y^2}\right)^{-1/2} \div \left(\frac{x^{-1}y^3}{x^2y^{-3}}\right)^{-1/3} = x^a y^b$, then $a + b + 1$ is

(a) 0 (b) 1 (c) -1 (d) 2

Solution. Simplify inside first: $\frac{x^4y^{-2}}{x^{-2}y^2} = x^6y^{-4}$, and $\frac{x^{-1}y^3}{x^2y^{-3}} = x^{-3}y^6$.

$$(x^6y^{-4})^{-1/2} = x^{-3}y^2, \qquad (x^{-3}y^6)^{-1/3} = xy^{-2}$$

$$\frac{x^{-3}y^2}{xy^{-2}} = x^{-4}y^4 \Rightarrow a = -4,\ b = 4,\ a + b + 1 = 1$$

Answer: (b) 1

Q. 9 • Roots and powers

If α, β are the roots of $x^2 - 96x + 256 = 0$, then $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is

(a) 1 (b) 2 (c) 3 (d) 4

Solution. Sum of roots $\alpha + \beta = 96$; product $\alpha\beta = 256$; both roots are positive.

$$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha^{3/8}\alpha^{5/8} + \beta^{3/8}\beta^{5/8}}{\alpha^{5/8}\beta^{5/8}} = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{96}{(2^8)^{5/8}} = \frac{96}{32} = 3$$

Answer: (c) 3

Q. 10 • Substitution

If $3^{x-1} + 3\left(\frac{1}{3}\right)^{x-2} = 10$, then x may have the value

(a) $\frac{1}{3}$ (b) 3 (c) 9 (d) None of these

Solution. $3^{x-1} = \frac{3^x}{3}$ and $3 \cdot 3^{2-x} = \frac{27}{3^x}$. Let $3^x = t > 0$:

$$\frac{t}{3} + \frac{27}{t} = 10 \Rightarrow t^2 - 30t + 81 = 0 \Rightarrow (t-27)(t-3) = 0$$

$t = 27 \Rightarrow x = 3$; $t = 3 \Rightarrow x = 1$. Among the options, $x = 3$.

Answer: (b) 3

3. Surds

Rationalisation

A surd is an irrational root such as $\sqrt{3}$. To clear a surd from a denominator, multiply and divide by the **conjugate**, using $(a+b)(a-b) = a^2 - b^2$.

$$\frac{1}{3+\sqrt{8}} = \frac{3-\sqrt{8}}{(3+\sqrt{8})(3-\sqrt{8})} = \frac{3-\sqrt{8}}{9-8} = 3-\sqrt{8}$$

Hinglish mein samjho

Agar denominator $\sqrt{n+1} - \sqrt{n}$ type ho, to answer bas plus wala form hai: $\frac{1}{\sqrt{n+1}-\sqrt{n}} = \sqrt{n+1} + \sqrt{n}$.

Isi trick se series cancel ho jaati hai.

Q. 11 • Telescoping surds

The value of $\sqrt{\frac{1}{2-\sqrt{3}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{2}-1}}$ is

(a) 1 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$

Solution. Rationalise every term:

$$\sqrt{(2+\sqrt{3}) - (\sqrt{3}+\sqrt{2}) + (\sqrt{2}+1)} = \sqrt{2+1} = \sqrt{3}$$

Answer: (c) $\sqrt{3}$

Q. 12 • Number of real roots

Find the number of real roots of $(\sqrt{5}+2)^{x^2-3}+(\sqrt{5}-2)^{x^2-3}=18$.
(a) 2 (b) 4 (c) 6 (d) 0

Solution. $(\sqrt{5}+2)(\sqrt{5}-2)=1$, so the two terms are reciprocals. Let $t=(\sqrt{5}+2)^{x^2-3}$:

$$t+\frac{1}{t}=18 \Rightarrow t^2-18t+1=0 \Rightarrow t=9\pm 4\sqrt{5}=(\sqrt{5}\pm 2)^2$$

Since $\sqrt{5}-2=(\sqrt{5}+2)^{-1}$, this means $t=(\sqrt{5}+2)^{\pm 2}$. So

$$x^2-3=\pm 2 \Rightarrow x^2=5 \text{ or } x^2=1 \Rightarrow x=\pm\sqrt{5},\ \pm 1$$

Answer: (b) 4

4. Componendo and Dividendo (C & D)

C & D

If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

Example: $\frac{7}{4}=\frac{14}{8}\Rightarrow\frac{7+4}{7-4}=\frac{14+8}{14-8}\Rightarrow\frac{11}{3}=\frac{22}{6}$ ✓

Proof. Add 1 to both sides of $\frac{a}{b}=\frac{c}{d}$: $\frac{a+b}{b}=\frac{c+d}{d}$ ...(i). Subtract 1: $\frac{a-b}{b}=\frac{c-d}{d}$ ...(ii). Dividing (i) by (ii) gives the result.

Q. 13 • C & D twice

If $\frac{a+2b+3c+6d}{-a-2b+3c+6d}=\frac{2a+b+6c+3d}{-2a-b+6c+3d}$, then
(a) $ac=bd$ (b) $ad=bc$ (c) $a+d=b+c$ (d) $c=\frac{ab}{d}$

Solution. Apply C & D. On each side, Num + Den and Num − Den give

$$\frac{6c+12d}{2a+4b}=\frac{12c+6d}{4a+2b}\Rightarrow\frac{c+2d}{a+2b}=\frac{2c+d}{2a+b}\Rightarrow\frac{c+2d}{2c+d}=\frac{a+2b}{2a+b}$$

Apply C & D in the form (Num + Den)/(Num − Den):

$$\frac{3c+3d}{-c+d}=\frac{3a+3b}{-a+b}\Rightarrow\frac{c+d}{d-c}=\frac{a+b}{b-a}$$

Apply once more: $\frac{2d}{2c}=\frac{2b}{2a}\Rightarrow\frac{d}{c}=\frac{b}{a}\Rightarrow ad=bc$.

Answer: (b) $ad=bc$

PadhoNa Active Recall #2 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Simplify $(2^3)^4 \cdot 2^{-5}$.
2. Rationalise $\dfrac{1}{\sqrt{7}-\sqrt{6}}$.
3. Write $\sqrt[3]{ab}$ using powers.
4. State the C & D rule.
5. Find x if $2^x + 2^x = 32$.

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series — **Basics clear? Speed bhi banao.**

JEE mein sawal aana alag baat hai, time par aana alag. **PadhoNa** Test Series ke chapter tests se speed aur accuracy dono build karo. Abhi join karo!

5. Inequalities and Intervals

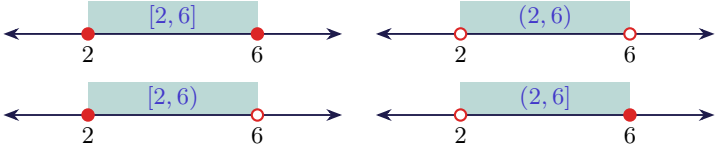
Inequality

Any expression involving $>$, $<$, $\geq$ or $\leq$ is called an inequality.

(i) $6 > 2$ True
(ii) $9 \geq 5$ True
(iii) $4 \geq 4$ True
(iv) $4 > 4$ False
(v) $6 \leq 11$ True
(vi) $6 \leq 6$ True

5.1 Intervals

Type	Inequality	Interval
Closed [ ]	$2 \leq x \leq 6$	$x \in [2, 6]$
Open ( )	$2 < x < 6$	$x \in (2, 6)$
Semi-open	$2 \leq x < 6$	$x \in [2, 6)$
Semi-open	$2 < x \leq 6$	$x \in (2, 6]$



More examples:

1. $x \geq 3 \Rightarrow x \in [3, \infty)$
2. $x > 3 \Rightarrow x \in (3, \infty)$
3. $x \leq 4 \Rightarrow x \in (-\infty, 4]$
4. $x < 0 \Rightarrow x \in (-\infty, 0)$
5. $-2 < x \leq 6 \Rightarrow x \in (-2, 6]$
6. $-3 \leq x < 0 \Rightarrow x \in [-3, 0)$
7. $-7 \leq x \leq 2 \Rightarrow x \in [-7, 2]$

Important notes

- ∞ ka matlab: jo sabse bada number soch sakte ho, usse bhi bada. Yeh koi number nahi hai, isliye ∞ aur $-\infty$ par hamesha **open bracket** lagega.
- $\frac{1}{0}$ is **not defined**; denominator zero kabhi allowed nahi.
- Curly bracket $\{\ \}$ matlab sirf woh values: $x \in \{2, 6\}$ means x is only 2 or 6, not the numbers in between.
- $\in$ ka matlab hai "belongs to".

Q. 14 • Counting integers

Find the number of integers in: (a) $(2, 8]$ (b) $(-5, 3)$ (c) $(0, 6]$ (d) $[-4, 4]$

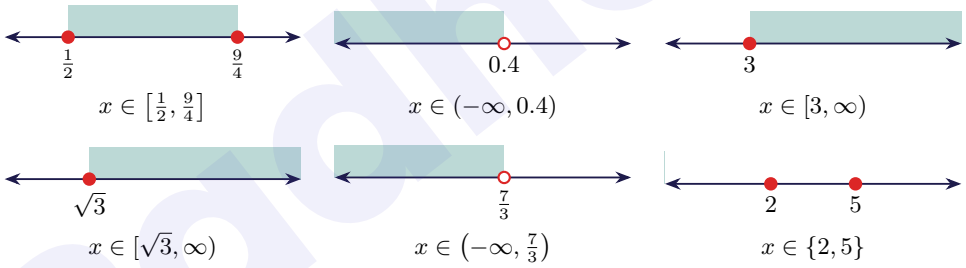
Solution. (a) $3, 4, 5, 6, 7, 8$: **6** integers.
(b) $-4, -3, -2, -1, 0, 1, 2$: **7** integers.
(c) $1, 2, 3, 4, 5, 6$: **6** integers.
(d) $-4, -3, \ldots, 3, 4$: **9** integers.

Answer: $6,\ 7,\ 6,\ 9$

5.2 Number line representation

Hinglish mein samjho

Open bracket matlab hollow dot (point shamil nahi). Closed bracket matlab solid dot (point shamil hai). Jis side ∞ ho, us side arrow chhod do.



6. Basic Rules for Solving Inequations

Rule 1. **Positive constant:** multiplying both sides by a positive constant keeps the sign same.
$$\frac{2x-3}{5} > 1 \Rightarrow 2x - 3 > 5 \Rightarrow 2x > 8 \Rightarrow x > 4.$$

Rule 2. **Negative constant:** multiplying both sides by a negative constant **reverses** the sign.
$$\frac{2x-3}{-5} > 1 \Rightarrow 2x - 3 < -5 \Rightarrow 2x < -2 \Rightarrow x < -1.$$

Rule 3. **Variable:** we **cannot** cross multiply by a variable, because its sign is unknown.
$\frac{2x-3}{x-1} > 1$ cannot be cross multiplied.

Galti mat karna

Same quantity dono taraf add ya subtract karne se sign nahi badalta. Negative se multiply ya divide karne par sign ulta. Variable se cross multiply karna sabse badi galti hai: wahan rational inequality wala method lagao.

Q. 15 • Chain inequality

Solve $3 \le 2x - 5 \le 7$ and write the solution in interval form.

Solution. Add 5 throughout: $8 \le 2x \le 12$. Divide by 2: $4 \le x \le 6$.

Answer: $x \in [4, 6]$

Q. 16 • Negative multiplier

Solve $8 \le -2(3x - 5) < 16$.

Solution. **Method 1.** Divide by -2, which reverses both signs:

$$-4 \ge 3x - 5 > -8 \;\Rightarrow\; 1 \ge 3x > -3 \;\Rightarrow\; \frac{1}{3} \ge x > -1$$

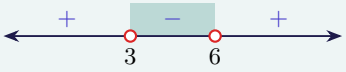
Method 2. Open the bracket: $8 \le -6x + 10 < 16$. Subtract 10: $-2 \le -6x < 6$. Divide by -6: $\frac{1}{3} \ge x > -1$.

Answer: $x \in \left(-1, \frac{1}{3}\right]$

7. Rational Inequalities and Wavy Curve Method

Q. 17 • Understanding by cases

Solve $\dfrac{x-3}{x-6} < 0$.



Solution. Critical points: $x = 3$ and $x = 6$. These split the line into three parts. Test one value in each:

- $x > 6$: take $x = 8$: $\dfrac{5}{2}$, positive.
- $3 < x < 6$: take $x = 4$: $\dfrac{1}{-2}$, negative.
- $x < 3$: take $x = 0$: $\dfrac{-3}{-6}$, positive.

We need negative, so $x \in (3, 6)$.

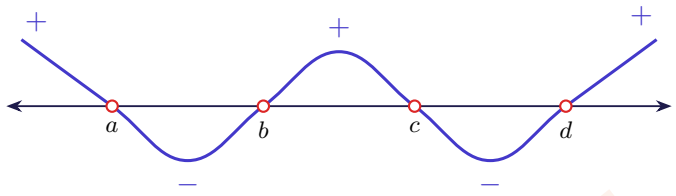
Answer: $x \in (3, 6)$

Hinglish mein samjho

Har baar values daal kar test karna lamba hai. Dekho, sign har critical point par badal raha hai! Isi pattern ka shortcut hai **Wavy Curve Method**: rightmost side hamesha plus (agar sab x ke coefficients positive hon), phir alternate.

7.1 Steps of the Wavy Curve Method

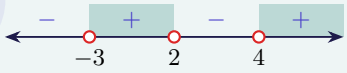
Step 1: Make the RHS zero and simplify into one fraction.
Step 2: Convert the LHS into linear factors.
Step 3: Make the coefficient of x positive in every linear factor.
Step 4: Equate each factor to zero to get critical points, and plot them on the number line.
Step 5: Starting from the rightmost end with $+$, write $+$ and $-$ alternately.
Step 6: Shade the required region.
Step 7: For $>$ or $<$: all critical points are open. For $\geq$ or $\leq$: numerator points are closed and denominator points are always open.



Curve upar matlab expression positive, curve neeche matlab negative. Rightmost part hamesha upar se shuru karo; har simple critical point par curve line cross karti hai.

Q. 18 • Three factors

Solve $\dfrac{(x+3)(x-2)}{x-4} > 0$.



Solution. Critical points: $-3, 2, 4$. All are open because of "$>$". Signs from the right: $+, -, +, -$.

Answer: $x \in (-3, 2) \cup (4, \infty)$

Q. 19 • Bring everything to one side

Solve $\dfrac{3x-1}{x+2} < 4$.



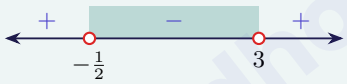
Solution. Never cross multiply! $\dfrac{3x-1}{x+2} - 4 < 0 \Rightarrow \dfrac{3x-1-4x-8}{x+2} < 0 \Rightarrow \dfrac{-(x+9)}{x+2} < 0.$

Multiply by -1 and flip: $\dfrac{x+9}{x+2} > 0$. Signs from right: $+, -, +$.

Answer: $x \in (-\infty, -9) \cup (-2, \infty)$

Q. 20 • Make the x coefficient positive

Solve $\dfrac{3x+5}{2x+1} > 2$.



Solution. $\dfrac{3x+5-2(2x+1)}{2x+1} > 0 \Rightarrow \dfrac{3-x}{2x+1} > 0 \Rightarrow \dfrac{-(x-3)}{2x+1} > 0 \Rightarrow \dfrac{x-3}{2x+1} < 0.$

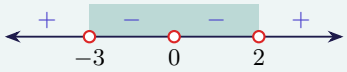
Answer: $x \in \left(-\frac{1}{2}, 3\right)$

Q. 21 • Cancelled factor

The set of real x for which $\dfrac{2x-x^2}{x^2+3x} > 0$ is

(a) $(-3, 2)$ (b) $(-3, 0)$

(c) $(-3, 2) - \{0\}$ (d) None of these



Solution. $\dfrac{x(2-x)}{x(x+3)} > 0$ with $x \neq 0$. Cancelling x: $\dfrac{-(x-2)}{x+3} > 0 \Rightarrow \dfrac{x-2}{x+3} < 0 \Rightarrow x \in (-3, 2)$. Remove 0, since the original fraction is undefined there.

Answer: (c) $(-3, 2) - \{0\}$

Galti mat karna

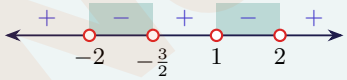
Cancel karne se pehle note kar lo ki woh factor zero nahi hona chahiye. Jo value denominator ko zero kare, woh answer se bahar hogi, chahe factor cancel ho gaya ho.

Q. 22 • Quadratics on both levels

The set of real x with $\dfrac{6x^2+3x-14}{x^2+x-2} < 4$ is

(a) $\left(-\frac{3}{2}, 2\right)$ (b) $\left(-2, -\frac{3}{2}\right) \cup (1, 2)$

(c) $(-2, 1)$ (d) None of these



Solution. $\dfrac{6x^2+3x-14-4(x^2+x-2)}{x^2+x-2} < 0 \Rightarrow \dfrac{2x^2-x-6}{x^2+x-2} < 0 \Rightarrow \dfrac{(2x+3)(x-2)}{(x+2)(x-1)} < 0.$

Answer: (b) $\left(-2, -\frac{3}{2}\right) \cup (1, 2)$

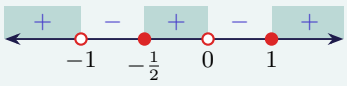
Q. 23 • Closed and open points

If S is the set of real x with $\dfrac{x+1}{x} - \dfrac{x-1}{x+1} \leq 2$, then S is

(a) $(-\infty, -1) \cup \left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

(b) $(-1, 0) \cup [1, \infty)$

(c) $\left[-\frac{1}{2}, 1\right]$ (d) None of these



Solution. $\dfrac{(x+1)^2 - x(x-1) - 2x(x+1)}{x(x+1)} \leq 0$. The numerator is $x^2 + 2x + 1 - x^2 + x - 2x^2 - 2x = -2x^2 + x + 1$.

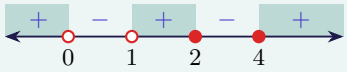
$$\frac{-(2x^2-x-1)}{x(x+1)} \leq 0 \Rightarrow \frac{(2x+1)(x-1)}{x(x+1)} \geq 0$$

Numerator points $-\frac{1}{2}, 1$ are closed; denominator points $-1, 0$ are open.

Answer: (a)

Q. 24 • Mixed

Solve $1 + \dfrac{3}{x-1} \geq \dfrac{8}{x}$.



Solution. $\dfrac{x(x-1)+3x-8(x-1)}{x(x-1)} \geq 0 \Rightarrow \dfrac{x^2-6x+8}{x(x-1)} \geq 0 \Rightarrow \dfrac{(x-2)(x-4)}{x(x-1)} \geq 0.$

Answer: $x \in (-\infty, 0) \cup (1, 2] \cup [4, \infty)$

PadhoNa Active Recall #3 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. In $[2, \infty)$, why is the bracket at ∞ open?
2. What happens to $<$ when you multiply by -3?
3. Solve $\dfrac{x-1}{x+5} < 0$.
4. For "$\geq$", which critical points are always open?
5. Number of integers in $(-2, 4]$?

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

7.2 Union and intersection of intervals

Meaning

Union $\cup$: **OR** (everything in either set). Intersection $\cap$: **AND** (only the common part).
Always draw intervals on a number line.

Example: take $A = \{1, 3, 5, 7\}$ and $B = \{3, 4, 5, 8\}$. Then $A \cup B = \{1, 3, 4, 5, 7, 8\}$ and $A \cap B = \{3, 5\}$.

Q. 25 • Intersection of unions

Find the intersection of:
$A = (-\infty, -3] \cup [1, 4] \cup [7, \infty)$
$B = \left[-6, -\frac{1}{2}\right] \cup [2, 8] \cup [9, 12]$



Solution. Draw both on parallel lines and keep the parts where both are present.

Answer: $[-6, -3] \cup [2, 4] \cup [7, 8] \cup [9, 12]$

Q. 26 • Double inequality

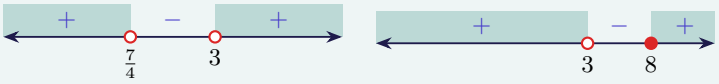
Solve $-2 < \dfrac{2x-1}{x-3} \leq 3$.

Solution. This means **both** conditions must hold.

First: $\dfrac{2x-1}{x-3} + 2 > 0 \Rightarrow \dfrac{4x-7}{x-3} > 0 \Rightarrow x \in \left(-\infty, \frac{7}{4}\right) \cup (3, \infty)$.

Second: $\dfrac{2x-1}{x-3} - 3 \leq 0 \Rightarrow \dfrac{-x+8}{x-3} \leq 0 \Rightarrow \dfrac{x-8}{x-3} \geq 0 \Rightarrow x \in (-\infty, 3) \cup [8, \infty)$.

Common part:



Answer: $x \in \left(-\infty, \frac{7}{4}\right) \cup [8, \infty)$

Q. 27 • Empty intersection

The solution set of $x^2 + 6 \leq 5x \leq 2x^2 - 7$ is

(a) ϕ (b) $[2, 3]$ (c) $(-\infty, -1] \cup \left[\frac{7}{2}, \infty\right)$ (d) None of these

Solution. **First part:** $x^2 - 5x + 6 \leq 0 \Rightarrow (x-2)(x-3) \leq 0 \Rightarrow x \in [2, 3]$.

Second part: $2x^2 - 5x - 7 \geq 0 \Rightarrow (2x-7)(x+1) \geq 0 \Rightarrow x \in (-\infty, -1] \cup \left[\frac{7}{2}, \infty\right)$.

Both must hold together, and these two sets have nothing in common.

Answer: (a) ϕ (empty set)

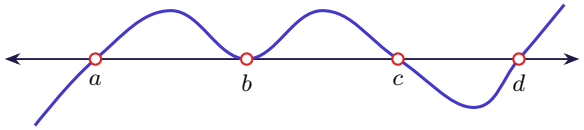
7.3 Wavy curve with repeated factors

Rule

Sign does **not** change at a critical point whose power is **even**. It changes when the power is odd.

Hinglish mein samjho

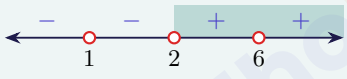
$(x-1)^4$ ya $(x-5)^2$ hamesha non-negative hote hain, isliye unke critical point par sign palatta nahi. Curve line ko chhookar wapas chali jaati hai. Bas dhyan rakho: even power wala point "> 0" mein answer se hatana padega, aur denominator mein ho to hamesha hategi.



Point b has even power:
curve touches, no sign change

Q. 28 • Even power in denominator

Solve $\dfrac{(x-2)^3(x-1)^4}{(x-6)^2} > 0$.

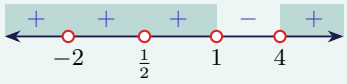


Solution. Odd power: $x = 2$ (sign changes). Even powers: $x = 1, 6$ (no change). From the right: $+, +, -, -$. Since we need > 0, $x = 1$ is out anyway, and $x = 6$ makes the denominator zero.

Answer: $x \in (2, 6) \cup (6, \infty)$

Q. 29 • Remove the even points

Solve $\dfrac{(x-1)^3(x-4)(2x-1)^2}{(x+2)^4} > 0.$



Solution. Odd powers at 1 and 4; even powers at -2 and $\frac{1}{2}$. From the right: $+$ (after 4), $-$ (between $1, 4$), then $+$ on all the rest. For strict > 0, the points -2 (denominator zero) and $\frac{1}{2}$ (numerator zero) are removed. Points $1, 4$ are open.

Answer: $x \in (-\infty, 1) \cup (4, \infty) - \{-2, \frac{1}{2}\}$

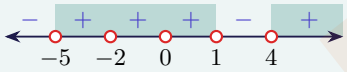
Q. 30 • Option type

The solution of $\dfrac{x^4(x+2)^2(x-1)}{(x-4)^3(x+5)} > 0$ is

(a) $(-5, 1) \cup (4, \infty)$ (b) $(-5, -2) \cup (-2, 0) \cup (0, 1) \cup (4, \infty)$

(c) $(-5, 0) \cup (0, 1)$ (d) $(-5, -2) \cup (4, \infty)$

Solution.



Odd power factors: $x = -5, 1, 4$. Even power factors: $x = -2, 0$. So the sign changes only at $-5, 1, 4$. For strict > 0, remove -2 and 0.

Answer: (b)

7.4 Wavy curve when a quadratic has $D < 0$

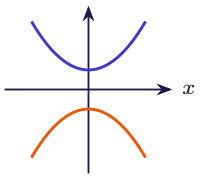
Quadratic with no real roots

For $ax^2 + bx + c$ with $D = b^2 - 4ac < 0$:

If $a > 0$, it is **always positive**.

If $a < 0$, it is **always negative**.

Such a factor has no critical point. Just remove it from the inequality, keeping track of its sign.



Q. 31 • Completing the idea

Solve $\dfrac{x^2 - 6x + 8}{x^2 + 2x + 3} < 0.$

Solution. Denominator: $a = 1$, $D = 4 - 12 = -8 < 0$, so it is always positive. Thus

$$(x-2)(x-4) < 0 \;\Rightarrow\; x \in (2, 4)$$

Alternatively: $x^2 + 2x + 3 = (x+1)^2 + 2$, which is clearly positive.

Answer: $x \in (2, 4)$

Q. 32 • Always negative denominator

Solve $\dfrac{2}{x - x^2 - 3} < 0.$

Solution. Denominator $-x^2+x-3$: $a = -1 < 0, D = 1-12 < 0$. So it is always negative, and positive/negative is always negative.

Answer: $x \in \mathbb{R}$

Q. 33 • Removing a positive factor

Solve $\dfrac{x^2 - 9x + 20}{3x^2 + 2x + 4} > 0.$

Solution. $3x^2 + 2x + 4$: $a > 0, D = 4 - 48 < 0$, always positive. So $(x - 4)(x - 5) > 0$.

Answer: $x \in (-\infty, 4) \cup (5, \infty)$

Q. 34 • Now cross multiply is safe

Solve $\dfrac{6x^2 - 1}{5x^2 - x + 2} < 1.$

Solution. Denominator: $a > 0, D = 1 - 40 < 0$, always positive, so we *may* cross multiply.

$$6x^2 - 1 < 5x^2 - x + 2 \;\Rightarrow\; x^2 + x - 3 < 0 \;\Rightarrow\; x = \frac{-1 \pm \sqrt{13}}{2}$$

Answer: $x \in \left(\frac{-1-\sqrt{13}}{2}, \frac{-1+\sqrt{13}}{2}\right)$

Q. 35 • Perfect square

Solve $\dfrac{x^2 + 8x - 14}{x^2 + 1} \leq 2.$

Solution. $x^2 + 1 > 0$ always. $x^2 + 8x - 14 \leq 2x^2 + 2 \Rightarrow x^2 - 8x + 16 \geq 0 \Rightarrow (x - 4)^2 \geq 0$, which is true for all real x.

Answer: $x \in \mathbb{R}$

PadhoNa Active Recall #4

Notes band karo. Yaad se likho.

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Sign change at a factor with even power?
2. Solve $\dfrac{(x-1)^2}{x+3} > 0.$
3. $x^2 + x + 1$ is always of which sign?
4. Union means OR or AND?
5. Intersection of $[1, 5]$ and $(3, 9)$?

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

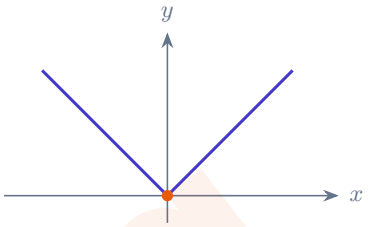
PadhoNa Test Series **Wavy curve is the key to Function chapters.**
Domain, range, calculus: sab jagah yeh lagega. **PadhoNa** Test Series se weekly practice karo aur sign wali galtiyan hamesha ke liye khatam karo.

8. Modulus (Absolute Value) Function

Definition

$$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Modulus gives the distance of x from 0 on the number line, so it can never be negative.



Hinglish mein samjho

Modulus ka matlab "number ko positive bana do" nahi hai. Sahi soch: **andar positive ho to waisa hi, negative ho to minus laga do.** Isliye $|x| = -x$ jab x negative ho, aur $-x$ tab positive hota hai.

Examples: $|-7| = -(-7) = 7$; $|4| = 4$; $|12| = 12$; $|2.5| = 2.5$; $|\sqrt{3}| = \sqrt{3}$; $|0| = 0$; $|-6| = 6$.

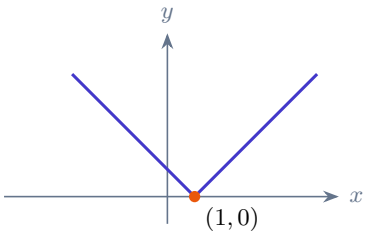
$$|x^2 - 7| = \begin{cases} x^2 - 7 & \text{if } x^2 - 7 \geq 0 \\ -(x^2 - 7) & \text{if } x^2 - 7 < 0 \end{cases}$$

8.1 Graphs of modulus functions

Example: $y = |x - 1|$.

$y = x - 1$ if $x \geq 1$: $\quad x = 1, 2, 3 \rightarrow y = 0, 1, 2.$
$y = 1 - x$ if $x < 1$: $\quad x = 0, -1, -2 \rightarrow y = 1, 2, 3.$

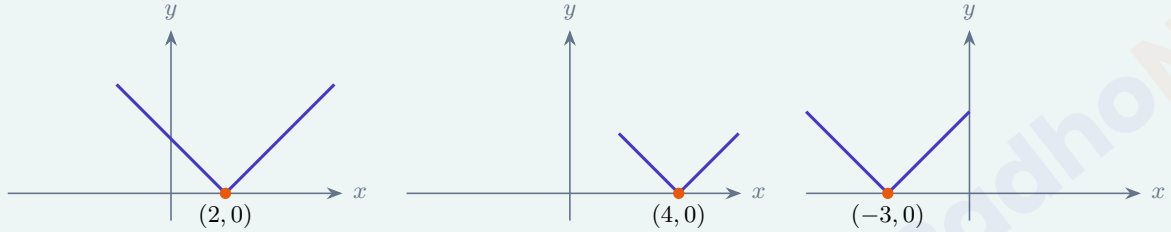
Plot these and join them. It is the same V shape, with its corner at $(1, 0)$.



Q. 36 • Draw the graphs

Draw (a) $y = |x - 2|$; (b) $y = |x - 4|$; (c) $y = |x + 3|$.

Solution. The corner is where the inside becomes zero.

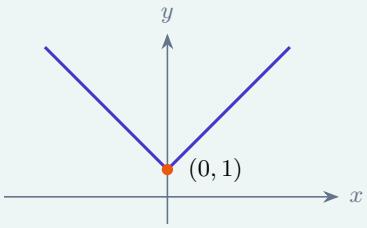


Q. 37 • Vertical shift by table

Draw $y = |x| + 1$.

Solution.

$y = x + 1$ if $x \geq 0$: $x = 0, 1 \to y = 1, 2$.
$y = -x + 1$ if $x < 0$: $x = -1, -2 \to y = 2, 3$.
Corner at $(0, 1)$.



Graphical transformations ($a > 0$)

$|x + a|$: shift $|x|$ by a units **left**. $|x - a|$: shift $|x|$ by a units **right**.
$|x| + a$: shift $|x|$ by a units **up**. $|x| - a$: shift $|x|$ by a units **down**.

8.2 Important properties of modulus

Properties

1. $|-x| = |x|$ **2.** $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ **3.** $|xy| = |x||y|$

4. $\sqrt{x^2} = |x|$ for $x \in \mathbb{R}$; also $x^2 = |x|^2$ **5.** $|x| \geq 0$

6. $|x + y| = |x| + |y| \iff xy \geq 0$ and $|x - y| = |x| + |y| \iff xy \leq 0$

Note on square roots: $\sqrt{36} = 6$ (only the positive value), $\sqrt{-9}$ is not real, $\sqrt{0} = 0$.

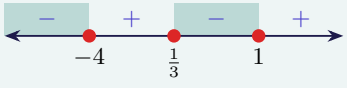
Why property 6 works. If x, y are both positive, $x + y$ is positive and both sides equal $x + y$. If both are negative, $|x+y| = -(x+y)$ and $|x| + |y| = -x - y$, again equal. If they have opposite signs, they partly cancel inside $|x+y|$, so equality fails. Zero is fine in either case.

Q. 38 • Brain Teaser

Solve $|x^2 + 3x - 4| + |1 - 3x| = |x^2 - 3|$.

Solution. Let $\alpha = x^2 + 3x - 4$ and $\beta = 1 - 3x$. Notice $\alpha + \beta = x^2 - 3$. So the equation is $|\alpha| + |\beta| = |\alpha + \beta|$, which holds exactly when $\alpha\beta \geq 0$.

$$(x + 4)(x - 1)(1 - 3x) \geq 0 \;\Rightarrow\; (x + 4)(x - 1)(3x - 1) \leq 0$$



Answer: $x \in (-\infty, -4] \cup \left[\frac{1}{3}, 1\right]$

8.3 Solving modulus equations

Basic: $|x| = 4 \Rightarrow x = \pm 4$; $|x| = \sqrt{5} \Rightarrow x = \pm\sqrt{5}$; $|x| = 0.3 \Rightarrow x = \pm 0.3$; $|x| = -2 \Rightarrow$ no solution, since modulus is never negative.

Single mod sign:

Q. 39 • Three basics

Solve: (a) $|x| = 3$; (b) $|x - 3| = 4$; (c) $\left|\dfrac{x-3}{2x-1}\right| = 4$.

Solution. (a) $x = \pm 3$.
(b) $x - 3 = \pm 4 \Rightarrow x = 7$ or $x = -1$.
(c) $\dfrac{x-3}{2x-1} = \pm 4$.
Positive: $x - 3 = 8x - 4 \Rightarrow 7x = 1 \Rightarrow x = \dfrac{1}{7}$.
Negative: $x - 3 = -8x + 4 \Rightarrow 9x = 7 \Rightarrow x = \dfrac{7}{9}$.

Answer: ± 3; $7, -1$; $\frac{1}{7}, \frac{7}{9}$

Q. 40 • Nested modulus

Solve $\big|\,||x-1|-2|-3\,\big| = 2$.

Solution. Open from outside: $||x-1|-2|-3 = \pm 2 \Rightarrow ||x-1|-2| = 5$ or 1.
Then $|x-1|-2 = \pm 5$ or $\pm 1 \Rightarrow |x-1| = 7, -3, 3, 1$.
$|x-1| = -3$ is rejected. The others give: $x = 1 \pm 7, x = 1 \pm 3, x = 1 \pm 1$.

Answer: $x = 8, -6, 4, -2, 2, 0$

Q. 41 • Sum of solutions

The sum of the solutions of $(x-2)^2 + |x-2| - 7 = 0$ is
(a) 3 (b) 4 (c) 5 (d) 6

Solution. Since $(x-2)^2 = |x-2|^2$, let $|x-2| = t \geq 0$: $t^2 + t - 7 = 0 \Rightarrow t = \dfrac{-1 \pm \sqrt{29}}{2}$.
The negative root is rejected. Let $a = \dfrac{\sqrt{29}-1}{2}$. Then $x - 2 = \pm a$, so $x_1 = 2 + a$ and $x_2 = 2 - a$, and $x_1 + x_2 = 4$.

Answer: (b) 4

Multiple mod signs:

Method

Find the point where each mod becomes zero; these split the number line into intervals. In each interval, open every mod with its correct sign, solve, and **accept the answer only if it lies in that interval.**

Q. 42 • Two mods

Solve $|x-2|+2|x-5|=13$.



Solution. **Case I,** $x<2$: $-(x-2)-2(x-5)=13 \Rightarrow -3x+12=13 \Rightarrow x=-\frac{1}{3}$. Lies in $x<2$. Accept.
Case II, $2\le x\le 5$: $(x-2)-2(x-5)=13 \Rightarrow -x+8=13 \Rightarrow x=-5$. Reject.
Case III, $x>5$: $(x-2)+2(x-5)=13 \Rightarrow 3x-12=13 \Rightarrow x=\frac{25}{3}$. Accept.

Answer: $x=-\frac{1}{3}, \frac{25}{3}$

Q. 43 • Counting solutions

Find the number of real x satisfying $|3x+1|+|4-x|=6x+5$.

Solution. Break points: $x=-\frac{1}{3}$ and $x=4$.
Case 1, $x<-\frac{1}{3}$: $-3x-1+4-x=6x+5 \Rightarrow -4x+3=6x+5 \Rightarrow x=-\frac{1}{5}$. But $-0.2>-0.33$, so reject.
Case 2, $-\frac{1}{3}\le x<4$: $3x+1+4-x=6x+5 \Rightarrow 2x+5=6x+5 \Rightarrow x=0$. Accept.
Case 3, $x\ge 4$: $3x+1-(4-x)=6x+5 \Rightarrow 4x-3=6x+5 \Rightarrow x=-4$. Reject.

Answer: One solution, $x=0$

Q. 44 • Three mods

Solve $|x-1|-2|x-4|+3|x+2|=12$.

Solution. Break points $-2, 1, 4$.
Case 1, $x\le -2$: $-x+1+2(x-4)-3(x+2)=12 \Rightarrow -2x-13=12 \Rightarrow x=-\frac{25}{2}$. Accept.
Case 2, $-2<x<1$: $-x+1+2(x-4)+3(x+2)=12 \Rightarrow 4x-1=12 \Rightarrow x=\frac{13}{4}$. Reject.
Case 3, $1\le x\le 4$: $x-1+2(x-4)+3(x+2)=12 \Rightarrow 6x-3=12 \Rightarrow x=\frac{5}{2}$. Accept.
Case 4, $x>4$: $x-1-2(x-4)+3(x+2)=12 \Rightarrow 2x+13=12 \Rightarrow x=-\frac{1}{2}$. Reject.

Answer: $x=-\frac{25}{2}, \frac{5}{2}$

Q. 45 • A whole interval

Solve $|x-1|+|x+3|=4$.

Solution. **Case 1,** $x\le -3$: $-2x-2=4 \Rightarrow x=-3$. Accept.
Case 2, $-3<x<1$: $-(x-1)+(x+3)=4 \Rightarrow 4=4$, which is always true. Every x in this case works.
Case 3, $x\ge 1$: $2x+2=4 \Rightarrow x=1$. Accept.

Answer: $x\in[-3,1]$

Hinglish mein samjho

Agar case mein variable ud jaye aur statement hamesha sach ho, to **poora case** answer hai. Hamesha jhooth ho to poora case reject. Geometry se bhi dekho: $|x-1|+|x+3|$ matlab x ki doori 1 aur -3 dono se; beech ke

har point par sum 4 hi rehta hai.

Q. 46 • With exponentials

The solution set of $2^x(2^x-1)+3=|2^x-2|+|2^x-3|$ contains
(a) one element (b) two elements (c) at least four elements (d) infinitely many

Solution. Let $2^x=t>0$: $t^2-t+3=|t-2|+|t-3|$.
Case 1, $t\le 2$: $t^2-t+3=5-2t\Rightarrow t^2+t-2=0\Rightarrow t=1$ or $t=-2$. Since $t>0$, $t=1$ and $x=0$.
Case 2, $2<t<3$: $t^2-t+3=1\Rightarrow t^2-t+2=0$, $D<0$: none.
Case 3, $t\ge 3$: $t^2-t+3=2t-5\Rightarrow t^2-3t+8=0$, $D<0$: none.

Answer: (a)

Q. 47 • Two mods with coefficients

Solve $|2x-3|+2|3x-7|=8$.

Solution. Break points $\frac{3}{2}$ and $\frac{7}{3}$.
Case 1, $x<\frac{3}{2}$: $-2x+3-6x+14=8\Rightarrow -8x=-9\Rightarrow x=\dfrac{9}{8}$. Accept.
Case 2, $\frac{3}{2}\le x\le\frac{7}{3}$: $2x-3-6x+14=8\Rightarrow -4x=-3\Rightarrow x=\dfrac{3}{4}$. Reject.
Case 3, $x>\frac{7}{3}$: $2x-3+6x-14=8\Rightarrow 8x=25\Rightarrow x=\dfrac{25}{8}$. Accept.

Answer: $x=\frac{9}{8},\ \frac{25}{8}$

Q. 48 • Sum of non-negatives

Solve $|x-2030|+|x-2031|+|x-2032|=0$.

Solution. Each term is non-negative, so the sum is zero only if all three are zero together: $x=2030$, $x=2031$ and $x=2032$ at once. That is impossible.

Answer: No solution

Q. 49 • Mod of a quadratic

The sum of the roots of $|x^2-7x+10|-2x+6=0$ is
(a) $\dfrac{17+\sqrt{17}}{2}$ (b) $\dfrac{9+\sqrt{17}}{2}$ (c) 9 (d) $\dfrac{17-\sqrt{17}}{2}$

Solution. $x^2-7x+10=(x-2)(x-5)$.
Case 1, $x\le 2$ **or** $x\ge 5$: $x^2-9x+16=0\Rightarrow x=\dfrac{9\pm\sqrt{17}}{2}\approx 6.56,\ 2.44$. Accept $\dfrac{9+\sqrt{17}}{2}$; reject 2.44, which lies in $(2,5)$.
Case 2, $2<x<5$: $-(x^2-7x+10)-2x+6=0\Rightarrow x^2-5x+4=0\Rightarrow x=1,4$. Accept 4; reject 1.
Sum $=\dfrac{9+\sqrt{17}}{2}+4=\dfrac{17+\sqrt{17}}{2}$.

Answer: (a)

PadhoNa Active Recall #5 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Write the definition of $|x|$.
2. Solve $|x+1| = -5$.
3. $\sqrt{x^2}$ equals?
4. $|a+b| = |a| + |b|$ holds when?
5. Graph of $|x+2|$ is $|x|$ shifted which way?
6. Solve $|2x-1| = 5$.

Score: _____ / 6 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

9. Modulus Inequalities

9.1 Single mod

For $a > 0$

$|x| \le a \Rightarrow x \in [-a, a]$ $\quad$ $|x| \ge a \Rightarrow x \in (-\infty, -a] \cup [a, \infty)$



$|x| \le 4$ $\quad$ $|x| \ge 6$

Q. 50 • Positive denominator

The set of x with $\dfrac{|x|-2}{|x|+3} > 0$ is

(a) $(-\infty, -3) \cup (3, \infty)$ (b) $(-3, -2) \cup (2, 3)$ (c) $(-\infty, -2) \cup (2, \infty)$ (d) $(-3, 3)$

Solution. $|x| + 3$ is always positive, so $|x| - 2 > 0 \Rightarrow |x| > 2$.

Answer: (c)

Q. 51 • Between two values

Solve (a) $2 < |x| < 6$; (b) $\dfrac{5}{2} < |x| \le 7$.

Solution. (a) $|x| > 2$ gives $(-\infty, -2) \cup (2, \infty)$; $|x| < 6$ gives $(-6, 6)$. Common part: $(-6, -2) \cup (2, 6)$.
(b) In the same way, the positive side is $\left(\frac{5}{2}, 7\right]$ and the negative side is its mirror image.

Answer: (a) $(-6, -2) \cup (2, 6)$; (b) $\left[-7, -\frac{5}{2}\right) \cup \left(\frac{5}{2}, 7\right]$

Hinglish mein samjho

$|x|$ wale sawal ka answer hamesha 0 ke around mirror image hota hai. Positive side nikaal lo, negative side uska ulta. Brackets ka type bhi mirror hota hai.

Q. 52 • Wavy curve in $|x|$

Solve $\dfrac{|x|-2}{(|x|-3)(|x|+1)} \le 0$.

Solution. $|x|+1>0$ always. Let $|x|=t$: $\dfrac{t-2}{t-3} \le 0 \Rightarrow t \in [2,3)$. So $2 \le |x| < 3$.

Answer: $x \in (-3,-2] \cup [2,3)$

Q. 53 • Shifted

Solve $2 < |x+1| \le 6$.

Solution. $2 < x+1 \le 6$ or $-6 \le x+1 < -2$.
Subtract 1: $1 < x \le 5$ or $-7 \le x < -3$.

Answer: $x \in [-7,-3) \cup (1,5]$

Q. 54 • Nested

If $\big||x-1|-4\big| \le 3$, the complete set of x is
(a) $[-6,8]$ (b) $[-6,0] \cup [2,8]$ (c) $[2,8]$ (d) $[-6,2] \cup [4,8]$

Solution. $-3 \le |x-1|-4 \le 3 \Rightarrow 1 \le |x-1| \le 7$.
So $1 \le x-1 \le 7$ or $-7 \le x-1 \le -1$, i.e. $2 \le x \le 8$ or $-6 \le x \le 0$.

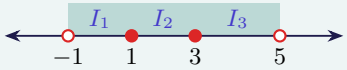
Answer: (b)

9.2 Multiple mods

Q. 55 • Sum of two mods, less than

Solve $|x-1|+|x-3|<6$.

Solution. **Case 1,** $x \le 1$: $4-2x<6 \Rightarrow x>-1$. So $I_1=(-1,1]$.
Case 2, $1<x\le 3$: $2<6$, always true. So $I_2=(1,3]$.
Case 3, $x>3$: $2x-4<6 \Rightarrow x<5$. So $I_3=(3,5)$.
Final answer is the union $I_1 \cup I_2 \cup I_3$.



Answer: $x \in (-1,5)$

Q. 56 • Greater than or equal

Solve $|x-2|+2|x+1| \ge 6$.

Solution. **Case 1,** $x \le -1$: $-(x-2)-2(x+1) \ge 6 \Rightarrow -3x \ge 6 \Rightarrow x \le -2$. So $I_1=(-\infty,-2]$.
Case 2, $-1<x<2$: $-(x-2)+2(x+1) \ge 6 \Rightarrow x+4 \ge 6 \Rightarrow x \ge 2$. Nothing in this case.
Case 3, $x \ge 2$: $(x-2)+2(x+1) \ge 6 \Rightarrow 3x \ge 6 \Rightarrow x \ge 2$. So $I_3=[2,\infty)$.

Answer: $x \in (-\infty,-2] \cup [2,\infty)$

PadhoNa Active Recall #6 **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Solve $|x| < 3$.
2. Solve $|x| > 3$.
3. Solve $|x - 2| \leq 1$.
4. Solve $1 \leq |x| \leq 2$.
5. Can $|x| < -1$ have a solution?

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series **Modulus done. Ab aata hai Log!**

Modulus aur Log JEE ke sabse scoring topics hain. **PadhoNa** Test Series ke tests mein inke tricky sawal practice karo aur exam mein aasani se marks lo.

10. Logarithm: Definition and Basic Properties

Hinglish mein samjho

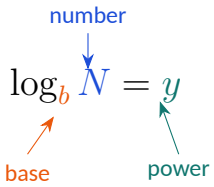
Logarithm ek hi sawal poochhta hai: **base ko kitni power doge ki number mil jaye?** $\log_2 16 = 4$ kyunki $2^4 = 16$. Bas! Log aur power ek hi cheez ko do tarike se likhna hai.

Definition

$$\log_b N = y \iff N = b^y$$

Number $=$ $(\text{base})^{\text{RHS}}$

Conditions: $N > 0,\ b > 0,\ b \neq 1$.



Examples:

- $\log_3 81 = y \Rightarrow 81 = 3^y \Rightarrow 3^4 = 3^y \Rightarrow y = 4$.
- $\log_4 256 = y \Rightarrow 256 = 4^y \Rightarrow y = 4$.
- $\log_{(-3)} 7$: not defined (negative base). $\log_3 0$: not defined (number must be positive).
- $\log_{\sqrt{5}} \sqrt{3}$: defined. $\log_1 7$: not defined (base 1). $\log_7 1$: defined, equals 0.
- $\log_2 7 = y$: here y is a real number, found using log tables.

Galti mat karna

$\log_1 1$ bhi **not defined** hai. Agar $\log_1 1 = y$, to $1 = 1^y$, jo har y ke liye sach hai, yaani koi unique value nahi. Isliye base kabhi 1 nahi hota.

10.1 Basic properties

Properties 1 to 6

P1. $\log_a a = 1$ **P2.** $\log_{1/a} a = -1$ **P3.** $\log_b 1 = 0$

P4. $\log_b(xy) = \log_b x + \log_b y$ **P5.** $\log_b \dfrac{x}{y} = \log_b x - \log_b y$ $(x, y > 0)$

P6. $\log_b x^y = y \log_b x$ (for $x > 0$)

Proofs.

- **P1:** Let $\log_a a = y$. Then $a^1 = a^y \Rightarrow y = 1$.
- **P2:** Let $\log_{1/a} a = y$. Then $a = \left(\frac{1}{a}\right)^y = a^{-y} \Rightarrow y = -1$.
- **P3:** Let $\log_b 1 = y$. Then $1 = b^y \Rightarrow y = 0$.
- **P4:** Let $\log_b(xy) = p$, $\log_b x = q$, $\log_b y = r$. Then $xy = b^p$, $x = b^q$, $y = b^r$. So $b^q b^r = b^p \Rightarrow q + r = p$. Similarly $\log_b(xyz) = \log_b x + \log_b y + \log_b z$.
- **P5:** Same method. In general, $\log_k \dfrac{ab}{cd} = \log_k a + \log_k b - \log_k c - \log_k d$.
- **P6:** $\log x^y = \log(x \cdot x \cdot x \cdots) = \log x + \log x + \cdots = y \log x$ (shown for natural y; the result holds for all real y).

Q. 57 • Simplify

Simplify:

(i) $\log_6 2 + \log_6 3$ (a) 5 (b) 1 (c) 6 (d) 2

(ii) $\log_{10} 50 + \log_{10} 20 - \log_{10} 100$ (a) 1 (b) 3 (c) 4 (d) 5

(iii) $\log_{35} \dfrac{21}{2} + \log_{35} \dfrac{20}{3} - \log_{35} 2$ (a) 3 (b) 2 (c) 0 (d) None

Solution. (i) $\log_6(2 \times 3) = \log_6 6 = 1$.

(ii) $\log_{10} \dfrac{50 \times 20}{100} = \log_{10} 10 = 1$.

(iii) $\log_{35}\left(\dfrac{21}{2} \times \dfrac{20}{3} \times \dfrac{1}{2}\right) = \log_{35} 35 = 1$, which is not among (a), (b), (c).

Answer: (i) (b); (ii) (a); (iii) (d)

Q. 58 • Inside out

Find the value of $\log_5 \left[\log_2\{\log_3(\log_2 512)\}\right]$.

(a) 1 (b) 2 (c) 3 (d) 0

Solution. Start from the innermost bracket: $\log_2 512 = 9$. Then $\log_3 9 = 2$. Then $\log_2 2 = 1$. Then $\log_5 1 = 0$.

Answer: (d) 0

Q. 59 • Matrix match

Match Column I with Column II (one item may match more than one).

	Column I		Column II
A	$\dfrac{\log_3 729}{\log_2 \sqrt{64}}$	p	positive integer
B	$\dfrac{2\log 10}{\log 20 + \log 5}$	q	negative integer
C	$\log_{1/2}\left(\dfrac{1}{8}\right)^{-2}$	r	rational but not integer
D	$\dfrac{\log_3 64 - \log_3 8}{\log_3 32}$	s	prime

Solution. **A:** $\log_3 729 = 6$; $\sqrt{64} = 8 = 2^3$, so $\log_2 \sqrt{64} = 3$. Value $= 2$: positive integer and prime. A $\to$ p, s.
B: $\dfrac{\log 100}{\log(20 \times 5)} = \dfrac{\log 100}{\log 100} = 1$. B $\to$ p.
C: $\left(\frac{1}{8}\right)^{-2} = \left(\frac{1}{2}\right)^{-6}$, so the value is -6. C $\to$ q.
D: $\dfrac{\log_3(64/8)}{\log_3 32} = \dfrac{\log_3 2^3}{\log_3 2^5} = \dfrac{3}{5}$. D $\to$ r.

Answer: A: p, s; B: p; C: q; D: r

10.2 Common mistakes while applying log

Galti mat karna

$\log_b(x+y) = \log_b x + \log_b y$	Wrong
$\dfrac{\log x}{\log 2} = 3 \Rightarrow \dfrac{x}{2} = 3$ (log cannot be cancelled like a number)	Wrong
$\dfrac{\log_b x}{\log_b y} = \log_b x - \log_b y$	Wrong
$(\log x)(\log y) = \log x + \log y$	Wrong
$\log_b \dfrac{x}{y} = \log_b x - \log_b y$	Right
$\log(xy) = \log x + \log y$	Right

Q. 60 • Prime factor bookkeeping

The value of $16\log\dfrac{16}{15} + 12\log\dfrac{25}{24} + 7\log\dfrac{81}{80}$ is
(a) $\log 2$ (b) $\log 3$ (c) $\log 5$ (d) None of these

Solution. Write every fraction in primes: $\dfrac{16}{15} = \dfrac{2^4}{3\cdot 5}$, $\dfrac{25}{24} = \dfrac{5^2}{2^3\cdot 3}$, $\dfrac{81}{80} = \dfrac{3^4}{2^4\cdot 5}$.

$$\log\left[\frac{2^{64}}{3^{16}5^{16}}\cdot\frac{5^{24}}{2^{36}3^{12}}\cdot\frac{3^{28}}{2^{28}5^{7}}\right]$$

Power of 2: $64 - 36 - 28 = 0$. Power of 3: $-16 - 12 + 28 = 0$. Power of 5: $-16 + 24 - 7 = 1$.
So the value is $\log 5$.

Answer: (c) $\log 5$

Q. 61 • $\sqrt{x^2} = |x|$

The value of $\sqrt{\log_{0.2}^2 125}$ is
(a) -3 (b) $\sqrt{-9}$ (c) 3 (d) None of these

Solution. $\sqrt{(\log_{0.2} 125)^2} = |\log_{0.2} 125|$. Since $0.2 = 5^{-1}$ and $125 = 5^3$, the log equals -3. So the value is $|-3| = 3$.

Answer: (c) 3

Q. 62 • Simplify the radical

Simplify $\log_{1/2} \sqrt[4]{1024 \cdot \sqrt[3]{4^{-1} \cdot 8^{-4/3}}}$.

Solution. Let N be the number. Write all in base 2: $4^{-1} = 2^{-2}$, $8^{-4/3} = 2^{-4}$.

$$N = \left[2^{10} \cdot (2^{-2} \cdot 2^{-4})^{1/3}\right]^{1/4} = \left[2^{10} \cdot 2^{-2}\right]^{1/4} = (2^8)^{1/4} = 4$$

$\log_{1/2} 4 = \log_{1/2} \left(\frac{1}{2}\right)^{-2} = -2$.

Answer: -2

Two most common bases

Base 10**:** $\log_{10} N$ (common log).
Base e**:** $\log_e x$, written $\ln x$ (natural log), where $e \approx 2.718$ is Napier's constant. For example, $\ln 2 = \log_e 2$.

10.3 Solving logarithm equations

Q. 63 • Basic forms

(a) Solve $\log_3 x = 4$. (b) Solve $|\log_3 x| = 2$.

Solution. (a) $x = 3^4 = 81$. (b) $\log_3 x = \pm 2 \Rightarrow x = 9$ or $x = \frac{1}{9}$.

Answer: 81; $9, \frac{1}{9}$

Q. 64 • Difference of logs

Find x: $\log_3(x+2) - \log_3(3x-4) = 0$.
(a) 1 (b) 2 (c) 3 (d) 4

Solution. $\log_3 \frac{x+2}{3x-4} = 0 \Rightarrow \frac{x+2}{3x-4} = 3^0 = 1 \Rightarrow x+2 = 3x-4 \Rightarrow x = 3$.
Check: $x+2 = 5 > 0$, $3x-4 = 5 > 0$ ✓

Answer: (c) 3

Q. 65 • Peel the layers

Solve $\log_{16}\left(2\log_2(1+\log_3(1+2\log_2 x))\right)=\frac{1}{4}$.

Solution. Let N be the whole inner bracket. $\log_{16} N=\frac{1}{4}\Rightarrow N=16^{1/4}=2$.
$2\log_2(1+\log_3(1+2\log_2 x))=2\Rightarrow\log_2(\dots)=1\Rightarrow 1+\log_3(1+2\log_2 x)=2$.
$\log_3(1+2\log_2 x)=1\Rightarrow 1+2\log_2 x=3\Rightarrow\log_2 x=1\Rightarrow x=2$.

Answer: $x=2$

Q. 66 • Take log on both sides

Solve $x^{\log_{10} x+2}=1000$.

Solution. Take $\log_{10}$ on both sides: $(\log_{10} x+2)\log_{10} x=3$. Let $\log_{10} x=t$:

$$t^2+2t-3=0\ \Rightarrow\ (t+3)(t-1)=0\ \Rightarrow\ t=1 \text{ or } t=-3$$

Answer: $x=10$ or $x=\frac{1}{1000}$

PadhoNa Active Recall #7 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Convert $\log_2 32=5$ to exponential form.
2. Three conditions for $\log_b N$ to exist?
3. $\log_{1/5} 5=$?
4. Is $\log(x+y)=\log x+\log y$?
5. Find $\log_4 8$ by writing both in base 2.
6. Solve $\log_5 x=-1$.

Score: _____ / 6 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

11. Advanced Properties of Logarithm

11.1 Property 7: Base change theorem

Base change

$$\log_b N=\frac{\log_c N}{\log_c b}=\frac{\log \text{ of number}}{\log \text{ of base}}\quad\text{(any valid base } c\text{)}$$

Examples: $\log_5 7=\dfrac{\log_3 7}{\log_3 5}$, $\log_9 11=\dfrac{\log_{10} 11}{\log_{10} 9}$.

Q. 67 • Product of logs

Solve: $(\log_3 8)(\log_4 3)$.

Solution. $\dfrac{\log 8}{\log 3}\cdot\dfrac{\log 3}{\log 4}=\dfrac{3\log 2}{2\log 2}=\dfrac{3}{2}$.

Answer: $\frac{3}{2}$

Q. 68 • Chain

The value of $\log_2 3\cdot\log_3 4\cdot\log_4 5\cdots\log_{15}16$ is
(a) 2 (b) 3 (c) 4 (d) 8

Solution. Base change makes it a chain: $\dfrac{\log 3}{\log 2}\cdot\dfrac{\log 4}{\log 3}\cdots\dfrac{\log 16}{\log 15}=\dfrac{\log 16}{\log 2}=4$.

Answer: (c) 4

11.2 Property 8: Reciprocal

Reciprocal

$$\log_b a=\frac{1}{\log_a b}\quad\text{i.e.}\quad \log_b a\cdot\log_a b=1$$

Proof. $\log_b a=\dfrac{\log_a a}{\log_a b}=\dfrac{1}{\log_a b}$. **Examples:** $\log_5 9=\dfrac{1}{\log_9 5}$, $\log_3 7=\dfrac{1}{\log_7 3}$.

Q. 69 • Substitution with base change

Solve for x: $1+2\log_{(x+1)}3=\log_3(x+1)$.

Solution. Let $\log_{(x+1)}3=t$. Then $\log_3(x+1)=\dfrac{1}{t}$.

$$1+2t=\frac{1}{t}\ \Rightarrow\ 2t^2+t-1=0\ \Rightarrow\ (2t-1)(t+1)=0$$

$t=\dfrac{1}{2}$: $3=(x+1)^{1/2}\Rightarrow x+1=9\Rightarrow x=8$.

$t=-1$: $3=(x+1)^{-1}\Rightarrow x+1=\dfrac{1}{3}\Rightarrow x=-\dfrac{2}{3}$.

Both give a valid base ($x+1>0$, $x+1\neq 1$).

Answer: $x=8,\ -\frac{2}{3}$

Q. 70 • Exact value

Find the exact value of $\dfrac{\log_2 24}{\log_{96} 2} - \dfrac{\log_2 192}{\log_{12} 2}$.

Solution. Using the reciprocal rule: $(\log_2 24)(\log_2 96) - (\log_2 192)(\log_2 12)$.
Let $\log_2 3 = t$: $\log_2 24 = 3 + t$, $\log_2 96 = 5 + t$, $\log_2 192 = 6 + t$, $\log_2 12 = 2 + t$.

$$(3+t)(5+t) - (6+t)(2+t) = (15 + 8t + t^2) - (12 + 8t + t^2) = 3$$

Answer: 3

11.3 Property 9: Base power formula

Base power

$\log_{a^k} m = \dfrac{1}{k} \log_a m$ General: $\log_{b^k} x^y = \dfrac{y}{k} \log_b x$

Proof. $\log_{a^k} m = \dfrac{\log_a m}{\log_a a^k} = \dfrac{\log_a m}{k \log_a a} = \dfrac{1}{k} \log_a m.$

Examples: $\log_{3^{50}} 3 = \dfrac{1}{50}$; $\log_{\sqrt{3}} 9 = \log_{3^{1/2}} 3^2 = \dfrac{2}{1/2} = 4.$

Q. 71 • Log to a surd base

The logarithm of $27\sqrt[5]{9}$ to the base $3\sqrt{3}$ is
(a) $\dfrac{34}{15}$ (b) $\dfrac{17}{5}$ (c) 3 (d) None of these

Solution. $27\sqrt[5]{9} = 3^3 \cdot 3^{2/5} = 3^{17/5}$; $3\sqrt{3} = 3^{3/2}$.

$$\log_{3^{3/2}} 3^{17/5} = \frac{17/5}{3/2} = \frac{34}{15}$$

Answer: (a) $\frac{34}{15}$

Q. 72 • In terms of m

If $\log_3 2 = m$, then $\log_9 12$ equals
(a) $2(1 + 2m)$ (b) $\dfrac{1 + 2m}{2}$ (c) $\dfrac{2}{1 + 2m}$ (d) $1 + m$

Solution. $\log_9 12 = \dfrac{1}{2} \log_3 12 = \dfrac{1}{2}(\log_3 3 + \log_3 2^2) = \dfrac{1}{2}(1 + 2m).$

Answer: (b)

11.4 Property 10 and 11: Power of a log

Very important

P10. $N = a^{\log_a N}$ **P11.** $a^{\log_b c} = c^{\log_b a}$

Also: if $\log_a x = \log_a y$, then $x = y$.

Proof of P10. Let $a^{\log_a N} = x$. Take $\log_a$: $(\log_a N)(\log_a a) = \log_a x \Rightarrow \log_a N = \log_a x \Rightarrow N = x$.

Examples: $3^{\log_3 7} = 7$; $9^{\log_9 11} = 11$; $2^{2\log_2 5} = 2^{\log_2 25} = 25$; $3^{\log_5 7} = 7^{\log_5 3}$; $6^{\log_2 5} = 5^{\log_2 6}$.

Q. 73 • Three terms

The value of $16^{1/\log_3 2} + 8^{\log_4 25} + 5^{4/\log_3 25}$ is

(a) 125 (b) 206 (c) 215 (d) 81

Solution. **First:** $16^{\log_2 3} = 2^{4\log_2 3} = 2^{\log_2 3^4} = 81$.

Second: $8^{\log_4 25} = 2^{3\cdot\frac{1}{2}\log_2 25} = 2^{\log_2 25^{3/2}} = 125$.

Third: $5^{4\log_{25} 3} = 5^{2\log_5 3} = 5^{\log_5 9} = 9$.

Sum $= 81 + 125 + 9 = 215$.

Answer: (c) 215

Q. 74 • Domain check

If $x^2 + 3^{\log_3 x} - 6 = 0$, then x is

(a) 2 (b) -3 (c) 3 (d) -2

Solution. For $\log_3 x$ to exist, $x > 0$. Then $3^{\log_3 x} = x$, so $x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) = 0$. Reject $x = -3$.

Answer: (a) 2

Q. 75 • Relation in a, b, c

If $a = \log_{40} 20$, $b = \log_{60} 40$ and $c = \log_{80} 60$, then abc equals

(a) $2bc - 1$ (b) $2bc + 1$ (c) $bc - 1$ (d) $bc + 1$

Solution. Using base change, $abc = \dfrac{\log 20}{\log 40} \cdot \dfrac{\log 40}{\log 60} \cdot \dfrac{\log 60}{\log 80} = \log_{80} 20$, and $bc = \log_{80} 40$.

$$2bc = \log_{80} 40^2 = \log_{80} 1600 = \log_{80}(80 \times 20) = 1 + \log_{80} 20$$

So $abc = 2bc - 1$.

Answer: (a)

Q. 76 • Brain Teaser

Solve for x: $9^{\log_{10} x} = 3 + 2 \cdot x^{\log_{10} 3}$.

Solution. By P11, $x^{\log_{10} 3} = 3^{\log_{10} x}$. Let $3^{\log_{10} x} = t > 0$. Then $9^{\log_{10} x} = t^2$.

$$t^2 = 3 + 2t \;\Rightarrow\; (t-3)(t+1) = 0 \;\Rightarrow\; t = 3 \text{ (reject } t = -1)$$

$3^{\log_{10} x} = 3^1 \Rightarrow \log_{10} x = 1 \Rightarrow x = 10$.

Answer: $x = 10$

Q. 77 • Brain Teaser: system

Solve for x, y, z ($a > 0, a \neq 1$):
$(\log_a x)\log_a(xyz) = 18,\ (\log_a y)\log_a(xyz) = 6,\ (\log_a z)\log_a(xyz) = 12.$

Solution. Add all three: $\log_a(xyz)\,[\log_a x + \log_a y + \log_a z] = 36$. The bracket is $\log_a(xyz)$ itself. So $[\log_a(xyz)]^2 = 36 \Rightarrow \log_a(xyz) = \pm 6$.
Case $+6$: $\log_a x = 3, \log_a y = 1, \log_a z = 2$.
Case -6: $\log_a x = -3, \log_a y = -1, \log_a z = -2$.

Answer: (a^3, a, a^2) **or** (a^{-3}, a^{-1}, a^{-2})

Q. 78 • Brain Teaser: number of solutions

The number of real solutions of $\log(-x) = 2\log(x+2)$ is
(a) 0 (b) 1 (c) 2 (d) 4

Solution. Domain: $-x > 0$ and $x + 2 > 0$, i.e. $-2 < x < 0$.
$\log(-x) = \log(x+2)^2 \Rightarrow -x = x^2 + 4x + 4 \Rightarrow x^2 + 5x + 4 = 0 \Rightarrow x = -1$ or $x = -4$.
Only $x = -1$ lies in the domain.

Answer: (b) 1

PadhoNa Active Recall #8 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Write the base change theorem.
2. $\log_2 9 \cdot \log_3 2 =$?
3. $5^{\log_5 3} =$?
4. $\log_{\sqrt{2}} 8 =$?
5. $2^{\log_3 7}$ equals 7 to which power?

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series — **Log ke saare rules aa gaye!**

Ab inhe mix karke sawal solve karna hai. **PadhoNa** Test Series mein aise hi mixed sawal milte hain. Practice karo, rank badhao.

12. Characteristic and Mantissa

Definition

For $\log_{10} N$, the integer part is the **characteristic** and the decimal part is the **mantissa**. If $\log_{10} N = 1.301$, the characteristic is 1 and the mantissa is 0.301.
The mantissa is always non-negative: $0 \leq m < 1$.

$\log_{10} 10 = 1$	$\log_{10} 100 = 2$	$\log_{10} 1000 = 3$	$\log_{10} 10000 = 4$
Characteristic 2 is for $100, 101, \ldots, 999$ (3 digit numbers).			
Characteristic 3 is for $1000, 1001, \ldots, 9999$ (4 digit numbers).			

Digits

If the characteristic of $\log_{10} N$ is x, then N has $x+1$ digits.
If the characteristic is $-n$, there are $n-1$ zeros right after the decimal point in N.

Examples: If $\log_{10} N$ has characteristic 4, N has 5 digits. Characteristic 30 means 31 digits. Also, for base 5: $\log_5 5 = 1$, $\log_5 25 = 2$, $\log_5 125 = 3$, and $\log_5 90$ has characteristic 2, since $25 < 90 < 125$.

Q. 79 • Counting with base 3

Find the number of positive integers which have characteristic 4 when the base of the logarithm is 3.

Solution. $\log_3 81 = 4$ and $\log_3 243 = 5$. So characteristic is 4 for $81, 82, \ldots, 242$.
Count $= 242 - 81 + 1 = 162$.

Answer: 162

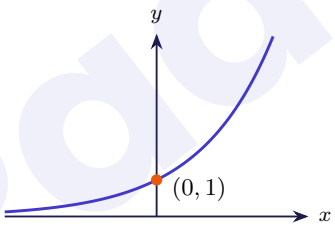
Q. 80 • Number of digits

Find the number of digits in 6^{200}. (Given $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$.)
(a) 155 (b) 156 (c) 157 (d) None of these

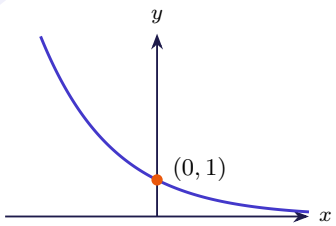
Solution. $\log_{10} 6^{200} = 200(\log_{10} 2 + \log_{10} 3) = 200(0.7781) = 155.62$.
Characteristic is 155, so the number of digits is $155 + 1 = 156$.

Answer: (b) 156

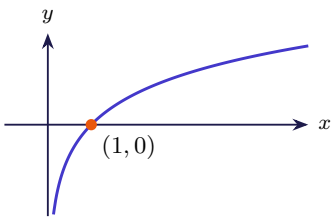
13. Graphs of Exponential and Log Functions



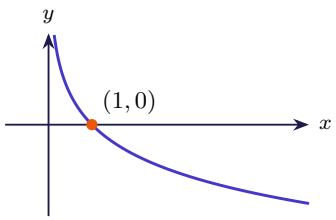
$y = a^x,\ a > 1$ (increasing)



$y = a^x,\ 0 < a < 1$ (decreasing)



$y = \log_a x,\ a > 1$ (increasing)



$y = \log_a x,\ 0 < a < 1$ (decreasing)

Sign of a log

Number and base on the **same side** of 1: log is **positive**. e.g. $\log_2 10 > 0$, $\log_{0.05} 0.01 > 0$.
Number and base on **opposite sides** of 1: log is **negative**. e.g. $\log_{0.2} 10 < 0$.

Hinglish mein samjho

Exponential graph hamesha x-axis ke upar rehta hai, kyunki a^x kabhi negative ya zero nahi hota. Log graph hamesha y-axis ke right mein, kyunki number positive hi hona chahiye. Dono ek-dusre ka mirror hain.

14. Logarithmic Inequalities

Sabse important rule

Log inequality mein **pehle domain nikalo**, aur final answer domain ke saath intersection karke hi likho. Yeh step chhoda to answer galat.

Two cases

Case 1: base is a constant, e.g. $\log_2(x-3) \geq 0$.
Case 2: base has a variable, e.g. $\log_{4x}(x-1) < 2$.

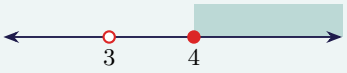
14.1 Case 1(A): constant base greater than 1

Base > 1

Sign of inequality **stays the same** when the log is removed, because $\log_a x$ is increasing.

Q. 81 • Domain plus inequality

Solve $\log_2(x-3) \geq 0$.

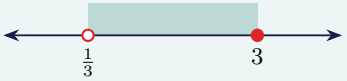


Solution. Domain: $x - 3 > 0 \Rightarrow x > 3$ ...(i).
Inequality: $x - 3 \geq 2^0 = 1 \Rightarrow x \geq 4$ ...(ii).
Intersection of (i) and (ii): $[4, \infty)$.

Answer: $x \in [4, \infty)$

Q. 82 • Upper bound

Solve $\log_2(3x-1) \leq 3$.



Solution. Domain: $3x - 1 > 0 \Rightarrow x > \dfrac{1}{3}$.
Inequality: $3x - 1 \leq 2^3 = 8 \Rightarrow 3x \leq 9 \Rightarrow x \leq 3$.

Answer: $x \in \left(\frac{1}{3}, 3\right]$

14.2 Case 1(B): constant base between 0 and 1

Base between 0 and 1

Sign of inequality **reverses** when the log is removed, because $\log_a x$ is decreasing.

Q. 83 • Reversal

Solve $\log_{1/2}(x-1) \geq 0$.



Solution. **Domain:** $x > 1$.

Inequality: $x - 1 \leq \left(\frac{1}{2}\right)^0 = 1 \Rightarrow x \leq 2$.

Answer: $x \in (1, 2]$

Q. 84 • Close decimals

Solve $\log_{1/3}(2x+1) \leq 2$.

Solution. **Inequality (sign reverses):** $2x + 1 \geq \left(\frac{1}{3}\right)^2 = \frac{1}{9} \Rightarrow 2x \geq -\frac{8}{9} \Rightarrow x \geq -\frac{4}{9} \approx -0.44$.

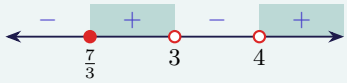
Domain: $2x + 1 > 0 \Rightarrow x > -\frac{1}{2} = -0.5$.

Since -0.44 is to the right of -0.5, the intersection is $x \geq -\frac{4}{9}$.

Answer: $x \in \left[-\frac{4}{9}, \infty\right)$

Q. 85 • Rational inside log

Solve $\log_2\left(\frac{2x^2 - 5x + 3}{x^2 - 7x + 12}\right) \geq 1$.



Solution. Base > 1: $\frac{2x^2 - 5x + 3}{x^2 - 7x + 12} \geq 2$. Any value ≥ 2 is automatically positive, so the domain is covered.

$$\frac{2x^2 - 5x + 3 - 2(x^2 - 7x + 12)}{x^2 - 7x + 12} \geq 0 \Rightarrow \frac{9x - 21}{(x-3)(x-4)} \geq 0 \Rightarrow \frac{3x - 7}{(x-3)(x-4)} \geq 0$$

Answer: $x \in \left[\frac{7}{3}, 3\right) \cup (4, \infty)$

Q. 86 • Between 0 and 1

Solve $\log_{1/3}(x^2 - 3) > 0$.

(a) $(-2, -\sqrt{3}) \cup (\sqrt{3}, 2)$ (b) $(-3, -2) \cup (2, 3)$ (c) $(-\infty, -2) \cup (2, \infty)$ (d) $(-2, 2)$

Solution. Sign reverses: $x^2 - 3 < \left(\frac{1}{3}\right)^0 = 1 \Rightarrow x^2 < 4$. Domain: $x^2 > 3$. Together $3 < x^2 < 4$.

Answer: (a)

14.3 Case 2: variable base

Hinglish mein samjho

Base mein x ho to pata nahi woh 1 se bada hai ya chhota. Isliye do cases banao: base > 1 aur $0 <$ base < 1. Dono ka answer nikalo, union lo, aur domain se intersection karo.

Q. 87 • Two cases

Solve $\log_{4x}(x^2 - 3x + 6) \geq 1$.

Solution. **Domain:** base $4x > 0$, $4x \neq 1$. Number $x^2 - 3x + 6$ has $D < 0$, $a > 0$: always positive.

Case 1, $4x > 1$**, i.e.** $x > \frac{1}{4}$**:** sign stays.

$$x^2 - 3x + 6 \geq 4x \;\Rightarrow\; x^2 - 7x + 6 \geq 0 \;\Rightarrow\; (x-1)(x-6) \geq 0$$

Combined with $x > \frac{1}{4}$: $I_1 = \left(\frac{1}{4}, 1\right] \cup [6, \infty)$.

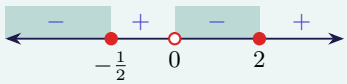
Case 2, $0 < x < \frac{1}{4}$**:** sign reverses. $(x-1)(x-6) \leq 0 \Rightarrow x \in [1, 6]$. Combined with $0 < x < \frac{1}{4}$: $I_2 = \phi$.

Answer: $x \in \left(\frac{1}{4}, 1\right] \cup [6, \infty)$

14.4 Log with substitution

Q. 88 • Let $\log = t$

Solve $\log_9 x - \log_x 9 \leq \frac{3}{2}$.



Solution. Domain: $x > 0$, $x \neq 1$. Let $\log_9 x = t \neq 0$. Then $\log_x 9 = \frac{1}{t}$.

$$t - \frac{1}{t} - \frac{3}{2} \leq 0 \;\Rightarrow\; \frac{2t^2 - 3t - 2}{2t} \leq 0 \;\Rightarrow\; \frac{(2t+1)(t-2)}{2t} \leq 0$$

So $t \in \left(-\infty, -\frac{1}{2}\right] \cup (0, 2]$. Converting back with base $9 > 1$:

$$x \in \left(0, 9^{-1/2}\right] \cup \left(9^0, 9^2\right]$$

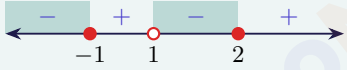
Answer: $x \in \left(0, \frac{1}{3}\right] \cup (1, 81]$

Q. 89 • Option type

Solve $\log_3 x \leq \dfrac{2}{\log_3 x - 1}$.

(a) $\left(0, \frac{1}{3}\right) \cup (3, 9)$ (b) $\left(0, \frac{1}{3}\right] \cup (3, 9]$

(c) $\left(\frac{1}{3}, 3\right) \cup [9, \infty)$ (d) None of these



Solution. Let $\log_3 x = t$: $t - \dfrac{2}{t-1} \leq 0 \Rightarrow \dfrac{t^2 - t - 2}{t-1} \leq 0 \Rightarrow \dfrac{(t-2)(t+1)}{t-1} \leq 0$.

$t \in (-\infty, -1] \cup (1, 2]$, so $x \in \left(0, 3^{-1}\right] \cup (3, 9]$.

Answer: (b)

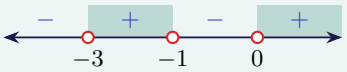
15. Exponential Inequalities

Rule

Base > 1: sign stays. Base between 0 and 1: sign reverses.

Q. 90 • Base greater than 1

Solve $5^{x+4} > \left(\frac{1}{125}\right)^{1/x}$.



Solution. $\left(\frac{1}{125}\right)^{1/x} = 5^{-3/x}$. Base $5 > 1$: $x + 4 > -\frac{3}{x}$.

$$x + 4 + \frac{3}{x} > 0 \Rightarrow \frac{x^2 + 4x + 3}{x} > 0 \Rightarrow \frac{(x+1)(x+3)}{x} > 0$$

Answer: $x \in (-3, -1) \cup (0, \infty)$

Q. 91 • Base less than 1

Solve $(0.2)^{1/x} \geq 0.008$.



Solution. $0.008 = (0.2)^3$. Base < 1, so sign reverses: $\frac{1}{x} \leq 3$.

$$\frac{1}{x} - 3 \leq 0 \Rightarrow \frac{1 - 3x}{x} \leq 0 \Rightarrow \frac{3x - 1}{x} \geq 0$$

Answer: $x \in (-\infty, 0) \cup \left[\frac{1}{3}, \infty\right)$

PadhoNa Active Recall #9

Notes band karo. Yaad se likho.

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. What must you do first in any log inequality?
2. $\log_{1/2} x > 2$ gives?
3. $\log_2 x < 3$ gives?
4. Solve $2^x \leq 8$.
5. Solve $\left(\frac{1}{2}\right)^x > 4$.

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

16. Square Root Inequations

Important

Squaring is allowed only when **both sides are non-negative**. Hamesha pehle domain nikalo.

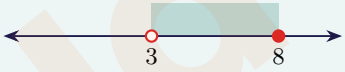
Why? $5 > 2 \Rightarrow 25 > 4$ ✓, but $2 > -5$ does **not** give $4 > 25$.
And $-1 > -4$ does not give $1 > 16$.

So when one side can be negative, make cases.

L	R	Square?
$+$	$+$	Yes
$-$	$+$	No
$+$	$-$	No
$-$	$-$	No (reverse)

Q. 92 • Both roots

Solve $\sqrt{3x-4} > \sqrt{8-x}$.



Solution. Both sides are non-negative, so we may square: $3x - 4 > 8 - x \Rightarrow x > 3$.

Domain: $3x - 4 \geq 0 \Rightarrow x \geq \frac{4}{3}$; and $8 - x \geq 0 \Rightarrow x \leq 8$.

Intersection with domain:

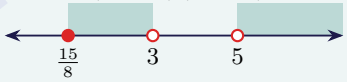
Answer: $x \in (3, 8]$

Q. 93 • Root less than x

Solve $\sqrt{8x-15} < x$.

Solution. **Domain:** $x \geq \frac{15}{8}$. So x is positive here, and both sides are non-negative.

Square: $8x - 15 < x^2 \Rightarrow x^2 - 8x + 15 > 0 \Rightarrow (x-3)(x-5) > 0 \Rightarrow x < 3$ or $x > 5$.



Answer: $x \in \left[\frac{15}{8}, 3\right) \cup (5, \infty)$

Q. 94 • RHS may be negative

Solve $\sqrt{5 + 4x - x^2} > 7 - 2x$.

Solution. **Domain:** $5 + 4x - x^2 \geq 0 \Rightarrow (x-5)(x+1) \leq 0 \Rightarrow x \in [-1, 5]$.

Case 1: RHS ≥ 0, i.e. $x \leq \frac{7}{2}$. Both sides are non-negative, so square:

$$5 + 4x - x^2 > 49 - 28x + 4x^2 \;\Rightarrow\; 5x^2 - 32x + 44 < 0 \;\Rightarrow\; (5x - 22)(x - 2) < 0 \;\Rightarrow\; 2 < x < \frac{22}{5}$$

With $x \leq \frac{7}{2}$: $I_1 = \left(2, \frac{7}{2}\right]$.

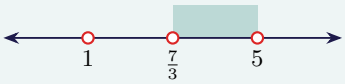
Case 2: RHS < 0, i.e. $x > \frac{7}{2}$. LHS is non-negative and RHS is negative, so it is always true. $I_2 = \left(\frac{7}{2}, \infty\right)$.

Union is $(2, \infty)$. Intersection with domain $[-1, 5]$:

Answer: $x \in (2, 5]$

Q. 95 • Concluding example

Solve $|2x-6| < x-1$.



Solution. LHS is non-negative, so RHS must be positive: $x > 1$.
Now both sides are non-negative. Square, using $|y|^2 = y^2$:

$$4x^2 - 24x + 36 < x^2 - 2x + 1 \;\Rightarrow\; 3x^2 - 22x + 35 < 0 \;\Rightarrow\; (3x-7)(x-5) < 0$$

So $x \in \left(\frac{7}{3}, 5\right)$, and this lies inside $x > 1$.

Answer: $x \in \left(\frac{7}{3}, 5\right)$

17. Solving Exponential Equations

Important note

If $|a|^b = 1$, then $b = 0$ (with $a \neq 0$) or $|a| = 1$. **Power zero or base one.**

Galti mat karna

0^0 is **not defined**. Jab power zero aaye, check karo base bhi zero to nahi ho gaya.

Q. 96 • Brain Teaser

The number of solutions of $|x-2|^{\frac{x^2-7x+10}{x-1}} = 1$ is

Solution. **Power** $= 0$: $x^2 - 7x + 10 = 0 \Rightarrow (x-2)(x-5) = 0 \Rightarrow x = 2$ or 5.
$x = 2$ gives 0^0: rejected. $x = 5$ gives $3^0 = 1$: accepted.
Base $= 1$: $|x-2| = 1 \Rightarrow x = 3$ or $x = 1$.
$x = 3$: accepted. $x = 1$ makes the denominator of the power zero: rejected.
So $x = 5$ and $x = 3$ are the only answers.

Answer: 2

Q. 97 • Brain Teaser

Find all solutions of $|x-2|^{(\log x)^2 - \log x^2} = |x-2|^8$, where the base of the logarithm is 10.

Solution. Domain: $x > 0$. Also $x = 2$ makes the base zero, with a negative power on the left, so it is rejected.
Dividing by $|x-2|^8$: $|x-2|^{(\log x)^2 - 2\log x - 8} = 1$.

Power $= 0$: let $\log x = t$: $t^2 - 2t - 8 = 0 \Rightarrow (t-4)(t+2) = 0 \Rightarrow t = 4$ or $t = -2$. So $x = 10^4$ or $x = \dfrac{1}{100}$.

Base $= 1$: $|x-2| = 1 \Rightarrow x = 3$ or $x = 1$. Both are positive, and both give $1 = 1$.

Answer: $x = 10^4,\ \frac{1}{100},\ 3,\ 1$

PadhoNa Active Recall #10

Notes band karo. Yaad se likho.

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Can we square $\sqrt{x} > -2$?
2. Solve $\sqrt{x-1} < 2$.
3. Is 0^0 defined?
4. $|a|^b = 1$ gives which two cases?
5. Solve $x^{x-3} = 1$ for $x > 0$.

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series

Chapter almost khatam!

Ek chapter theek se karna aadhi jeet hai. Ab **PadhoNa** Test Series ka full chapter test do aur dekho kitna yaad reh gaya.

PadhoNa

18. Short Notes and Master Formula Sheet

Identities

$(a \pm b)^2 = a^2 + b^2 \pm 2ab$
$a^2 - b^2 = (a - b)(a + b)$
$(a + b + c)^2 = \sum a^2 + 2\sum ab$
$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
$(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$
$a^3 + b^3 + c^3 - 3abc = (a + b + c)\left(\sum a^2 - \sum ab\right)$
$a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

Wavy curve

Denominator zero points are never in the answer.
For $\geq, \leq$: numerator points closed, denominator points open.
Cross multiply only if the denominator is surely positive.
Square only if both sides are surely non-negative.
Multiplying by a negative number flips the sign.
Adding or subtracting the same quantity keeps the sign.
Even power: no sign change.

Modulus

$|x| = x$ if $x \geq 0$; $-x$ if $x < 0$
$\sqrt{x^2} = |x|,\ |xy| = |x||y|,\ x^2 = |x|^2$
For $a > 0$: if $|x| < a$ then $x \in (-a, a)$
For $a > 0$: if $|x| > a$ then $x \in (-\infty, -a) \cup (a, \infty)$
$|x + y| = |x| + |y| \iff xy \geq 0$
$|x - y| = |x| + |y| \iff xy \leq 0$

Logarithm

P1 $\log_N N = 1$ **P2** $\log_{1/N} N = -1$
P3 $\log_a 1 = 0$
P4 $\log_a(MN) = \log_a M + \log_a N$ $(M, N > 0)$
P5 $\log_a \frac{m}{n} = \log_a m - \log_a n$ $(m, n > 0)$
P6 $\log_a m^x = x \log_a m$ $(m > 0)$
P7 $\log_b m = \dfrac{\log_a m}{\log_a b}$
P8 $\log_b a \cdot \log_a b = 1$
P9 $\log_{a^k} m = \frac{1}{k} \log_a m$
P10 $N = a^{\log_a N}$
P11 $a^{\log_b c} = c^{\log_b a}$

Log facts

Same side of 1: log positive.
Opposite sides of 1: log negative.
Mantissa: $0 \leq m < 1$.
Characteristic n of $\log_{10} N$: N has $n + 1$ digits.
Characteristic $-n$: $n - 1$ zeros after the decimal point in N.

Exponential function

$a^x > 0$ for all x $(a > 0)$.
$a > 1$: a^x increases with x.
$0 < a < 1$: a^x decreases with x.

Exponential inequalities

$$a^x > 1 \Rightarrow \begin{cases} x > 0 & a > 1 \\ x < 0 & 0 < a < 1 \end{cases}$$

$$a^x < 1 \Rightarrow \begin{cases} x < 0 & a > 1 \\ x > 0 & 0 < a < 1 \end{cases}$$

$$a^x > a^y \Rightarrow \begin{cases} x > y & a > 1 \\ x < y & 0 < a < 1 \end{cases}$$

$$a^x < a^y \Rightarrow \begin{cases} x < y & a > 1 \\ x > y & 0 < a < 1 \end{cases}$$

For $b > 0$:

$$a^x > b \Rightarrow \begin{cases} x > \log_a b & a > 1 \\ x < \log_a b & 0 < a < 1 \end{cases}$$

$$a^x < b \Rightarrow \begin{cases} x < \log_a b & a > 1 \\ x > \log_a b & 0 < a < 1 \end{cases}$$

With $\geq, \leq$, the results use $\geq, \leq$ in the same way.

Log inequalities

Base > 1: sign stays.
Base in $(0, 1)$: sign reverses.
Always intersect with the domain.

19. Active Recall Answer Key

Pehle khud attempt kiya? Tabhi yeh page dekho. Jo galat hua, us topic par wapas jao.

Active Recall #1

1. $(a+b)(a^2+b^2-ab)$. **2.** $36-8=28$. **3.** $3abc$. **4.** $8+18=26$. **5.** $a^2+b^2+c^2+2ab+2bc+2ca$.

Active Recall #2

1. 2^7. **2.** $\sqrt{7}+\sqrt{6}$. **3.** $a^{1/3}b^{1/3}$. **4.** $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{a-b}=\frac{c+d}{c-d}$. **5.** $2^{x+1}=2^5$, so $x=4$.

Active Recall #3

1. ∞ is not a number, so it can never be included. **2.** It becomes $>$. **3.** $(-5,1)$. **4.** Denominator points. **5.** 6.

Active Recall #4

1. No change. **2.** $(-3,\infty)-\{1\}$. **3.** Positive ($a>0$, $D<0$). **4.** OR. **5.** $(3,5]$.

Active Recall #5

1. x if $x\geq 0$; $-x$ if $x<0$. **2.** No solution. **3.** $|x|$. **4.** $ab\geq 0$. **5.** 2 units left. **6.** $x=3,-2$.

Active Recall #6

1. $(-3,3)$. **2.** $(-\infty,-3)\cup(3,\infty)$. **3.** $[1,3]$. **4.** $[-2,-1]\cup[1,2]$. **5.** No.

Active Recall #7

1. $2^5=32$. **2.** $N>0, b>0, b\neq 1$. **3.** -1. **4.** No. **5.** $\frac{3}{2}$. **6.** $x=\frac{1}{5}$.

Active Recall #8

1. $\log_b N=\frac{\log_c N}{\log_c b}$. **2.** 2. **3.** 3. **4.** 6. **5.** $\log_3 2$.

Active Recall #9

1. Find the domain. **2.** $0<x<\frac{1}{4}$. **3.** $0<x<8$. **4.** $x\leq 3$. **5.** $x<-2$.

Active Recall #10

1. No; one side is negative. It is true for all $x\geq 0$. **2.** $[1,5)$. **3.** No. **4.** $b=0$ or $|a|=1$. **5.** $x=3$ or $x=1$.

Chapter khatam. Ab shuru hota hai asli game!

Notes se samajh aati hai. Test se confidence aata hai. Apni speed, accuracy aur weak topics jaanne ke liye **PadhoNa Test Series** abhi join karo aur har chapter ke baad khud ko check karo.

Padho samajh ke. Yaad karo khud se. Test do roz se.