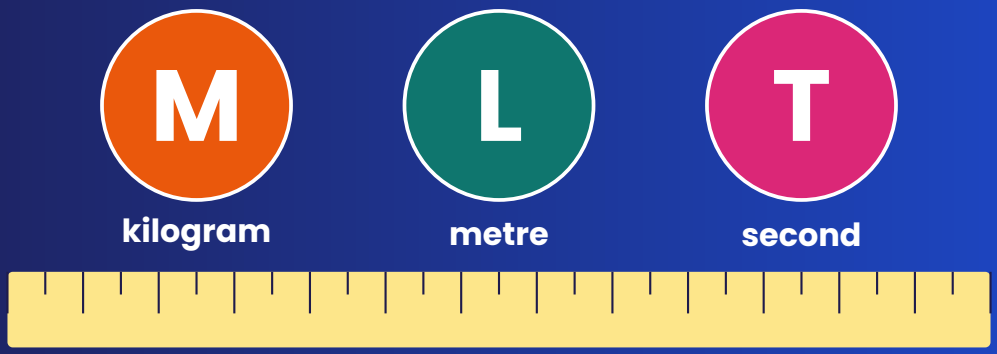
PadhoNa

Padho samajh ke. Yaad karo khud se.

PHYSICS • CLASS 11 • CHAPTER 1

Units and Measurements

Complete notes from basics to advanced
for Class 11, JEE Main, JEE Advanced and NEET



Simple Hinglish explanations • Colour diagrams • Fully solved questions
Active Recall quizzes • Master formula sheet

How to use these notes

Padhna aur **yaad karna** do alag cheezein hain. Sirf page palatne se lagta hai sab aa gaya, par exam mein dimaag blank ho jata hai. Isliye har topic ko is order mein karo:

1. **Concept padho**, phir uska **Hinglish** box padho taaki baat dimaag mein baith jaye.
2. **Solved question** ko pehle khud try karo. Solution tabhi dekho jab do minute try kar liya ho.
3. **Active Recall** box aate hi notes band karo aur copy mein likho. Answers last mein milenge.
4. Hafte ke end mein **Master Formula Sheet** se revise karo aur **PadhoNa Test Series** se apna level check karo.

Concept

Blue box: definition aur main theory.

Hinglish mein samjho

Yellow box: wahi baat simple Hinglish mein.

Yaad rakho: Formula

Green box: formula jo yaad rakhna hai.

Galti mat karna

Red box: common galtiyan.

PadhoNa Shortcut

Pink box: shortcut aur fast method.

Purple box: Active Recall. Yahan rukna compulsory hai!

Short forms: DF = dimensional formula. M, L, T stand for mass, length and time. Square brackets such as $[\mathrm{ML}]$ mean *the dimension of*.

What's inside

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1. Physical Quantities

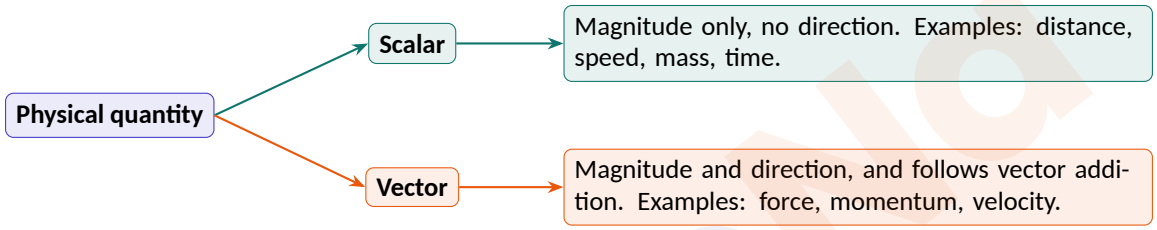
Hinglish mein samjho

Yeh chapter poori physics ki bhasha hai. Shuru mein tables dekh ke ghabrana mat. Sirf kuch formulas samajh lo, baaki sab apne aap nikal aayega. End tak yeh tumhara sabse scoring chapter ban sakta hai.

Concept

Anything that can be measured is a **physical quantity**: length, mass, time, force, work and so on. Beauty or honesty cannot be measured, so they are not physical quantities.

1.1 Classification by direction



PadhoNa Shortcut

Vector ki laws abhi yaad karne ki zaroorat nahi. Unke liye alag chapter hai. Abhi sirf itna samjho: direction ho aur vector addition follow kare, tabhi vector.

1.2 Classification by dependency

Concept

Fundamental (base) quantities are completely independent of one another, and every other quantity can be written in terms of them.
Derived quantities are built from the base quantities. Examples: speed = distance/time, volume, acceleration, force, pressure.

No.	Base quantity	SI unit	Symbol	Dimension
1	Length	metre	m	$[\mathrm{L}]$
2	Mass	kilogram	kg	$[\mathrm{M}]$
3	Time	second	s	$[\mathrm{T}]$
4	Temperature	kelvin	K	$[\mathrm{K}]$
5	Electric current	ampere	A	$[\mathrm{A}]$
6	Luminous intensity	candela	cd	$[\mathrm{cd}]$
7	Amount of substance	mole	mol	$[\mathrm{mol}]$

Yaad karne ka tareeka

Pehle teen: **MLT**. Phir **Temperature**, **Ampere**. Last mein **candela** aur **mole**. Kul saat.

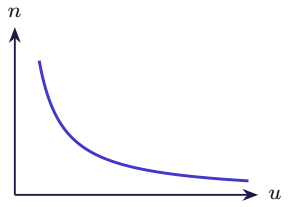
1.3 Numerical value and unit

Every quantity is a number multiplied by a unit. Changing the unit changes the number, but the quantity itself stays the same:

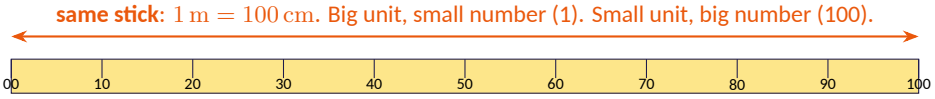
Yaad rakho: Formula

$$Q = n_1 u_1 = n_2 u_2$$

n = numerical value, u = size of the unit.



So $n \propto \frac{1}{u}$: **a bigger unit gives a smaller number**.



1.4 Systems of units

Type	Quantity	CGS	MKS	FPS
Fundamental	Length	cm	m	ft
Fundamental	Mass	g	kg	lb
Fundamental	Time	s	s	s
Derived	Force	dyne	newton	poundal
Derived	Work or energy	erg	joule	ft-poundal
Derived	Power	erg/s	watt	ft-poundal/s

The metre and kilogram came from France. CGS was formalised by the British Association in 1874, MKS was developed further by Giorgi in 1901, and FPS is the British system. SI is the modern extension of MKS.

Q.1 • Density conversion

(a) Convert $80\,\mathrm{kg/m^3}$ into g/cm^3. **(b)** Convert $1.2\,\mathrm{g/cm^3}$ into kg/m^3.

Solution. **(a)** $\dfrac{80 \times 1000\,\mathrm{g}}{(100\,\mathrm{cm})^3} = \dfrac{8 \times 10^4}{10^6} = 0.08\,\mathrm{g/cm^3}$.

(b) $\dfrac{1.2 \times 10^{-3}\,\mathrm{kg}}{(10^{-2}\,\mathrm{m})^3} = \dfrac{1.2 \times 10^{-3}}{10^{-6}} = 1200\,\mathrm{kg/m^3}$.

Answer: (a) $0.08\,\mathrm{g/cm^3}$ **(b)** $1200\,\mathrm{kg/m^3}$

PadhoNa Shortcut

Water ki density yaad rakho: $1\,\mathrm{g/cm^3} = 1000\,\mathrm{kg/m^3}$. To g/cc se kg/m^3 mein jaane ke liye 1000 se multiply karo, aur ulta jaane ke liye 1000 se divide.

2. Dimensions

Concept

The **dimensions** of a quantity are the powers to which the base quantities must be raised to represent it. Writing a quantity this way, in square brackets, gives its **dimensional formula (DF)**. The brackets remind us that we are comparing kinds of quantity, not their sizes.

Unit alag, dimension alag

Length ka **unit** m (metre) hai, aur mass ka **dimension** M hai. Chhota m aur capital M mix mat karna.

Q. 2 • Writing a DF

Write the dimensional formula of acceleration. What is the dimension of mass, of length and of time in it?

Solution. $[a] = \dfrac{[\text{velocity}]}{[\text{time}]} = \dfrac{[\mathrm{LT}^{-1}]}{[\mathrm{T}]} = [\mathrm{LT}^{-2}] = [\mathrm{M}^0\mathrm{L}^1\mathrm{T}^{-2}]$.
So the dimensions are 0 in mass, 1 in length and -2 in time.

Answer: $[\mathrm{M}^0\mathrm{LT}^{-2}]$

Hinglish mein samjho

School exam mein $[\mathrm{M}^0\mathrm{LT}^{-2}]$ pura likho, bracket ke saath, taaki number na kate. JEE ya mock test mein rough work karte waqt bracket likhne mein time mat lagao. Sirf LT^{-2} kaafi hai.

Q. 3 • Powers in a DF

Find the dimensions of mass, length and time in pressure.

Solution. $[P] = \dfrac{[F]}{[A]} = \dfrac{[\mathrm{MLT}^{-2}]}{[\mathrm{L}^2]} = [\mathrm{ML}^{-1}\mathrm{T}^{-2}]$. So they are $+1$, -1 and -2.

Answer: $1, -1, -2$

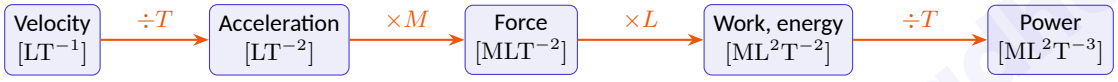
2.1 Base units and derived units

Concept

Base units are the units of base quantities. SI has seven of them.

Derived units are built from base units. Example: unit of acceleration $= \dfrac{\text{m/s}}{\text{s}} = \text{m/s}^2$.

2.2 Do not mug up: build the ladder



Hinglish mein samjho

Tables ratne mein time waste mat karo. Agar velocity, acceleration, force, momentum, work, power aur kinetic energy pakke hain, to baaki sab formula se 10 second mein nikal aayega. Tables sirf check karne aur revise karne ke liye hain.

2.3 Table: mechanics

Quantity	Formula	SI unit	Dimension
Velocity, speed (v)	$v = s/t$	m/s	$[\mathrm{M^0\,L\,T^{-1}}]$
Acceleration (a)	$a = \Delta v/\Delta t$	m/s^2	$[\mathrm{M^0\,L\,T^{-2}}]$
Momentum (p)	$p = mv$	kg m/s	$[\mathrm{M\,L\,T^{-1}}]$
Impulse (J)	$J = F\Delta t$	N s	$[\mathrm{M\,L\,T^{-1}}]$
Force (F)	$F = ma$	newton (N)	$[\mathrm{M\,L\,T^{-2}}]$
Pressure (P)	$P = F/A$	pascal (Pa)	$[\mathrm{M\,L^{-1}\,T^{-2}}]$
Kinetic energy (K)	$K = \frac{1}{2}mv^2$	joule (J)	$[\mathrm{M\,L^2\,T^{-2}}]$
Power (P)	$P = W/t$	watt (W) = J/s	$[\mathrm{M\,L^2\,T^{-3}}]$
Density (ρ)	$\rho = m/V$	kg/m^3	$[\mathrm{M\,L^{-3}\,T^0}]$
Angular displacement (θ)	$\theta = \text{arc}/\text{radius}$	radian (rad)	$[\mathrm{M^0\,L^0\,T^0}]$
Angular velocity (ω)	$\omega = \Delta\theta/\Delta t$	rad/s	$[\mathrm{M^0\,L^0\,T^{-1}}]$
Angular acceleration (α)	$\alpha = \Delta\omega/\Delta t$	rad/s^2	$[\mathrm{M^0\,L^0\,T^{-2}}]$

Quantity	Formula	SI unit	Dimension
Moment of inertia (I)	$I = mr^2$	kg m^2	$[\mathrm{M\,L^2\,T^0}]$
Torque (τ)	$\tau = Fr_\perp$	N m	$[\mathrm{M\,L^2\,T^{-2}}]$
Angular momentum (L)	$L = mvr$	J s	$[\mathrm{M\,L^2\,T^{-1}}]$
Spring constant (k)	$F = kx$	N/m	$[\mathrm{M\,L^0\,T^{-2}}]$
Gravitational constant (G)	$F = Gm_1m_2/r^2$	N m^2/kg^2	$[\mathrm{M^{-1}\,L^3\,T^{-2}}]$
Universal gas constant (R)	$PV = nRT$	J/(mol K)	$[\mathrm{M\,L^2\,T^{-2}\,K^{-1}\,mol^{-1}}]$
Planck's constant (h)	$E = h\nu$	J s	$[\mathrm{M\,L^2\,T^{-1}}]$
Surface tension (S)	$S = F/l$	N/m or J/m^2	$[\mathrm{M\,L^0\,T^{-2}}]$
Coefficient of viscosity (η)	$F = 6\pi\eta rv$	kg/(m s) = Pa s	$[\mathrm{M\,L^{-1}\,T^{-1}}]$
Time period (T)	$T = 1/\nu$	second (s)	$[\mathrm{M^0\,L^0\,T}]$
Frequency (ν)	$\nu = 1/T$	hertz (Hz)	$[\mathrm{M^0\,L^0\,T^{-1}}]$

2.4 Table: heat and thermodynamics

Quantity	Formula	SI unit	Dimension
Heat (Q)	form of energy	joule (J)	$[\mathrm{M\,L^2\,T^{-2}}]$
Specific heat (c)	$c = Q/(m\Delta T)$	J/(kg K)	$[\mathrm{M^0\,L^2\,T^{-2}\,K^{-1}}]$
Thermal (heat) capacity	$C = Q/\Delta T$	J/K	$[\mathrm{M\,L^2\,T^{-2}\,K^{-1}}]$
Latent heat (L)	$L = Q/m$	J/kg	$[\mathrm{M^0\,L^2\,T^{-2}}]$
Boltzmann constant (k_B)	$E_{\text{trans}} = \frac{3}{2}k_BT$	J/K	$[\mathrm{M\,L^2\,T^{-2}\,K^{-1}}]$
Thermal conductivity (k)	$\frac{Q}{t} = \frac{kA\Delta T}{d}$	J/(m s K)	$[\mathrm{M\,L\,T^{-3}\,K^{-1}}]$
Stefan's constant (σ)	$E = \sigma AT^4t$	W/(m^2 K^4)	$[\mathrm{M\,L^0\,T^{-3}\,K^{-4}}]$
Wien's constant (b)	$b = \lambda_{\max}T$	m K	$[\mathrm{M^0\,L\,T^0\,K}]$
Linear expansion coefficient (α)	$\alpha = \frac{\Delta l}{l\,\Delta T}$	K^{-1}	$[\mathrm{M^0\,L^0\,T^0\,K^{-1}}]$
Mechanical equivalent of heat (J)	$J = W/Q$	joule/calorie	$[\mathrm{M^0\,L^0\,T^0}]$
van der Waals a (for 1 mole)	a/V^2 adds to pressure	N m^4	$[\mathrm{M\,L^5\,T^{-2}}]$
van der Waals b (for 1 mole)	b subtracts from volume	m^3	$[\mathrm{M^0\,L^3\,T^0}]$
Temperature (T)	base quantity	kelvin (K)	$[\mathrm{M^0\,L^0\,T^0\,K}]$

2.5 Table: electricity and magnetism

Quantity	Formula	SI unit	Dimension
Electric charge (q)	$q = It$	coulomb (C)	$[\mathrm{M^0\,L^0\,T\,A}]$
Electric current (I)	$I = q/t$	ampere (A)	$[\mathrm{M^0\,L^0\,T^0\,A}]$
Capacitance (C)	$C = q/V$	farad (F)	$[\mathrm{M^{-1}\,L^{-2}\,T^4\,A^2}]$
Electric potential (V)	$V = W/q$	volt (V)	$[\mathrm{M\,L^2\,T^{-3}\,A^{-1}}]$
Permittivity of free space (ε_0)	$F = \frac{q_1q_2}{4\pi\varepsilon_0r^2}$	C^2/(N m^2)	$[\mathrm{M^{-1}\,L^{-3}\,T^4\,A^2}]$
Dielectric constant (K)	$K = \varepsilon/\varepsilon_0$	no unit	$[\mathrm{M^0\,L^0\,T^0}]$
Resistance (R)	$R = V/I$	ohm (Ω)	$[\mathrm{M\,L^2\,T^{-3}\,A^{-2}}]$
Resistivity (ρ)	$\rho = RA/l$	Ω m	$[\mathrm{M\,L^3\,T^{-3}\,A^{-2}}]$
Self inductance (L)	$\phi = LI$	henry (H)	$[\mathrm{M\,L^2\,T^{-2}\,A^{-2}}]$
Magnetic flux (ϕ)	$\phi = BA$	weber (Wb)	$[\mathrm{M\,L^2\,T^{-2}\,A^{-1}}]$

Quantity	Formula	SI unit	Dimension
Magnetic field (B)	$F = qvB$	tesla (T)	$[\mathrm{M\,L^0\,T^{-2}\,A^{-1}}]$
Magnetising field (H)	$H = B/\mu$	A/m	$[\mathrm{M^0\,L^{-1}\,T^0\,A}]$
Magnetic dipole moment (m)	$m = IA$	A m^2	$[\mathrm{M^0\,L^2\,T^0\,A}]$
Permeability of free space (μ_0)	$B = \dfrac{\mu_0 I}{2\pi r}$	N/A^2	$[\mathrm{M\,L\,T^{-2}\,A^{-2}}]$
Surface charge density (σ)	$\sigma = q/A$	C/m^2	$[\mathrm{M^0\,L^{-2}\,T\,A}]$
Electric dipole moment (p)	$p = qd$	C m	$[\mathrm{M^0\,L\,T\,A}]$
Conductance (G)	$G = 1/R$	siemens (S)	$[\mathrm{M^{-1}\,L^{-2}\,T^3\,A^2}]$
Conductivity (σ)	$\sigma = 1/\rho$	S/m	$[\mathrm{M^{-1}\,L^{-3}\,T^3\,A^2}]$
Current density (J)	$J = I/A$	A/m^2	$[\mathrm{M^0\,L^{-2}\,T^0\,A}]$
Electric field (E)	$E = F/q$	N/C = V/m	$[\mathrm{M\,L\,T^{-3}\,A^{-1}}]$
Rydberg constant (R_∞)	$\dfrac{1}{\lambda} = R_\infty\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$	m^{-1}	$[\mathrm{M^0\,L^{-1}\,T^0}]$

Electric wale DF ka shortcut

Do cheezein yaad karo: $[q] = [\mathrm{AT}]$ aur $[V] = \dfrac{\text{energy}}{\text{charge}} = [\mathrm{ML^2T^{-3}A^{-1}}]$. Isse $R = V/I$, $C = q/V$, $E = F/q$ sab nikal jayenge.

2.6 Table: other useful quantities

Quantity	Formula	SI unit	Dimension
Stress	F/A	N/m^2	$[\mathrm{M\,L^{-1}\,T^{-2}}]$
Strain	$\Delta l/l$	no unit	$[\mathrm{M^0\,L^0\,T^0}]$
Young's, bulk and shear modulus	stress/strain	N/m^2	$[\mathrm{M\,L^{-1}\,T^{-2}}]$
Wave intensity (I)	P/A	W/m^2	$[\mathrm{M\,L^0\,T^{-3}}]$
Wavelength (λ)	a length	m	$[\mathrm{M^0\,L\,T^0}]$
Gravitational potential	$V = -GM/r$	J/kg	$[\mathrm{M^0\,L^2\,T^{-2}}]$

2.7 Quantities with the same dimensions

Dimension	Quantities that share it
$[\mathrm{M^0\,L^0\,T^{-1}}]$	frequency, angular frequency, angular velocity, velocity gradient, decay constant
$[\mathrm{M\,L^2\,T^{-2}}]$	work, energy of every kind, heat, torque
$[\mathrm{M\,L^{-1}\,T^{-2}}]$	pressure, stress, all elastic moduli, energy density
$[\mathrm{M\,L\,T^{-1}}]$	momentum, impulse
$[\mathrm{M\,L\,T^{-2}}]$	force, weight, thrust
$[\mathrm{M\,L^2\,T^{-1}}]$	angular momentum, Planck's constant
$[\mathrm{M\,L^0\,T^{-2}}]$	surface tension, surface energy per area, spring constant
$[\mathrm{M^0\,L^2\,T^{-2}}]$	latent heat, gravitational potential
$[\mathrm{M\,L^2\,T^{-2}\,K^{-1}}]$	heat capacity, Boltzmann constant, entropy
$[\mathrm{M^0\,L^0\,T}]$	time, L/R, RC, $\sqrt{LC}$
$[\mathrm{M^0\,L\,T^0}]$	distance, displacement, radius, wavelength, radius of gyration
$[\mathrm{M^0\,L\,T^{-1}}]$	speed, velocity, speed of light
$[\mathrm{M^0\,L\,T^{-2}}]$	acceleration, gravity g, centripetal acceleration

Do pakke cheezein

Gas constant R per mole hota hai, isliye uski dimension mein mol^{-1} aata hai. Boltzmann constant aur heat capacity mein nahi aata.
Decay constant ki dimension $[\mathrm{T}^{-1}]$ hai, kyunki woh "per second" hota hai.

PadhoNa Active Recall #1 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Name the seven SI base quantities with their units.
2. If the unit of a quantity becomes 4 times bigger, what happens to its number?
3. Without looking, write the DF of force, work and power.
4. Give the DF of momentum and of angular momentum.
5. Convert $2\,\mathrm{g/cm^3}$ to kg/m^3.
6. Name a quantity with the same dimensions as torque.
7. Give the DF of electric charge and of voltage.

Score: _____ / 7 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series — **Base strong, chapter strong**

DF yaad se likh paaye? Ab inhe sawalon mein lagao. **PadhoNa Test Series** ke timed tests se dekho tum kitni speed se solve kar paate ho.

3. Principle of Homogeneity

Concept

In a correct physical equation, all terms that are added, subtracted or equated must have the **same dimensions**.

$$A = B + C - D$$

all four have the same DF

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

each term has DF $[\mathrm{LT}^{-1}]$

Hinglish mein samjho

Aam mein aam hi judta hai. 5 metre mein 3 second nahi jod sakte. Jo cheezein plus, minus ya equal se judi hain, un sab ki dimension same honi chahiye. Is chapter ke aadhe sawal yahin se solve ho jate hain.

3.1 Rules of dimensional analysis

- Quantities of different dimensions cannot be added or subtracted, but they can be multiplied or divided.
- Each additive term on either side must have the same DF.
- Trigonometric ratios, exponentials and logarithms are dimensionless.
- Multiplying by a pure number does not change the DF.

Q. 4 • Same DF, can we add?

Work and torque have the same dimensions. Can we add them?

Solution. No. Same dimensions is necessary for adding, but not enough. Work and torque are different kinds of quantity, so adding them has no physical meaning. Same DF does not always mean same physical quantity.

Answer: No

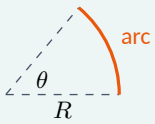
Q. 5 • Strain and angle

Find the unit and dimension of **(a)** strain, **(b)** plane angle.

Solution.

(a) Strain $= \dfrac{\Delta l}{l} = \dfrac{L}{L}$. So it has no unit and no dimension: $[\mathrm{M^0L^0T^0}]$.

(b) $\theta = \dfrac{\text{arc}}{\text{radius}} = \dfrac{L}{L}$. It is dimensionless, but its unit is the **radian**.



Answer: (a) no unit, no dimension (b) radian, dimensionless

Q. 6 • True or false

A physical quantity with no dimension must also have no unit.

Solution. False. Plane angle (radian) and solid angle (steradian) are dimensionless, but they do have units.

Answer: False

4. Applications of Dimensional Analysis

1. Find the DF of an unknown quantity	2. Check if a formula is right	3. Derive a relation between quantities	4. Convert units from one system to another

Most questions from this chapter come from these four uses.

4.1 Finding the DF of an unknown quantity

Q. 7 • Gravitational constant

Newton's law of gravitation is $F = \dfrac{Gm_1m_2}{r^2}$. Find the DF of G.

Solution. $[G] = \dfrac{[F][r^2]}{[m_1m_2]} = \dfrac{[\mathrm{MLT^{-2}}][\mathrm{L^2}]}{[\mathrm{M^2}]} = [\mathrm{M^{-1}L^3T^{-2}}]$.

Answer: $[\mathrm{M^{-1}L^3T^{-2}}]$

Q. 8 • Viscosity

The viscous force between layers of a liquid is $F = \eta A\dfrac{\mathrm{d}v}{\mathrm{d}y}$, where A is area and $\dfrac{\mathrm{d}v}{\mathrm{d}y}$ is velocity gradient. Find the DF of η.

Solution. $\left[\dfrac{\mathrm{d}v}{\mathrm{d}y}\right] = \dfrac{[\mathrm{LT^{-1}}]}{[\mathrm{L}]} = [\mathrm{T^{-1}}]$. So $[\eta] = \dfrac{[\mathrm{MLT^{-2}}]}{[\mathrm{L^2}][\mathrm{T^{-1}}]} = [\mathrm{ML^{-1}T^{-1}}]$.

Answer: $[\mathrm{ML^{-1}T^{-1}}]$

Q. 9 • Heat

Find the DF of **(a)** specific heat c, from $Q = mc\Delta T$; **(b)** latent heat L, from $Q = mL$.

Solution. **(a)** $[c] = \dfrac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{M}][\mathrm{K}]} = [\mathrm{L^2T^{-2}K^{-1}}]$. **(b)** $[L] = \dfrac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{M}]} = [\mathrm{L^2T^{-2}}]$.

Answer: **(a)** $[\mathrm{L^2T^{-2}K^{-1}}]$ **(b)** $[\mathrm{L^2T^{-2}}]$

Q. 10 • Boltzmann constant

The average translational kinetic energy of a gas molecule is $E = \dfrac{3}{2}k_BT$. Find the DF of k_B.

Solution. $\dfrac{3}{2}$ is a pure number, so $[k_B] = \dfrac{[E]}{[T]} = [\mathrm{ML^2T^{-2}K^{-1}}]$.

Answer: $[\mathrm{ML^2T^{-2}K^{-1}}]$

Q. 11 • Electric chain

Find the DF of **(a)** charge, **(b)** electric potential, **(c)** resistance, using heating power $P = I^2R$, **(d)** resistivity, from $R = \dfrac{\rho l}{A}$.

Solution. **(a)** $q = It$: $[\mathrm{AT}]$. **(b)** $V = \dfrac{W}{q}$: $\dfrac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{AT}]} = [\mathrm{ML^2T^{-3}A^{-1}}]$.

(c) $R = \dfrac{P}{I^2}$: $\dfrac{[\mathrm{ML^2T^{-3}}]}{[\mathrm{A^2}]} = [\mathrm{ML^2T^{-3}A^{-2}}]$. **(d)** $\rho = \dfrac{RA}{l}$: $[R][\mathrm{L}] = [\mathrm{ML^3T^{-3}A^{-2}}]$.

Answer: as above

Q. 12 • Unknown constant

The potential energy of a particle is $U = \beta x^3$, where x is distance. Find the DF of β.

Solution. $[\beta] = \dfrac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{L^3}]} = [\mathrm{ML^{-1}T^{-2}}]$.

Answer: $[\mathrm{ML^{-1}T^{-2}}]$

4.2 Checking whether a formula is correct

Q. 13 • Wrong formula

Is $F = \dfrac{mv^2}{r^2}$ dimensionally correct? (r is a radius)

Solution. LHS: $[\mathrm{MLT^{-2}}]$. RHS: $\dfrac{[\mathrm{M}][\mathrm{L^2T^{-2}}]}{[\mathrm{L^2}]} = [\mathrm{MT^{-2}}]$. They differ, so it is wrong.

Answer: Incorrect

Q. 14 • Right formula

For a planet in orbit, is $T^2 = \dfrac{4\pi^2 r^3}{GM}$ dimensionally correct?

Solution. LHS: $[\mathrm{T^2}]$. RHS: $\dfrac{[\mathrm{L^3}]}{[\mathrm{M^{-1}L^3T^{-2}}][\mathrm{M}]} = [\mathrm{T^2}]$. They match, so it is correct dimensionally.

Answer: Correct

Q. 15 • Test a relation

Check if $P = Fv^2$ is dimensionally correct, where P is power, F force and v velocity.

Solution. LHS: $[\mathrm{ML^2T^{-3}}]$. RHS: $[\mathrm{MLT^{-2}}][\mathrm{L^2T^{-2}}] = [\mathrm{ML^3T^{-4}}]$. They differ, so it is wrong. (The right form is $P = Fv$.)

Answer: Incorrect

Three statements to remember

- Dimensionally **wrong** $\Rightarrow$ the formula is certainly wrong.
- Dimensionally **right** $\Rightarrow$ the formula may be right or wrong.
- A **correct** formula is always dimensionally right.

Hinglish mein samjho

Example: $K = mv^2$ dimensionally bilkul sahi hai, phir bhi galat formula hai, kyunki asli formula $\frac{1}{2}mv^2$ hai. Number $\frac{1}{2}$ dimension analysis pakad hi nahi sakta.

4.3 Using homogeneity to find constants

Q. 16 • Polynomial in time

The velocity of a particle is $v = p + qt + rt^2 + st^3$. Find the dimensions of p, q, r, s.

Solution. Each term must have the DF of velocity.
$[p] = [\mathrm{LT}^{-1}]$, $[q] = [\mathrm{LT}^{-2}]$, $[r] = [\mathrm{LT}^{-3}]$, $[s] = [\mathrm{LT}^{-4}]$.

Answer: as above

Q. 17 • Three terms

For force $F = \dfrac{A}{t} + Bt^2 + Ct$, where t is time, find the DF of A, B and C.

Solution. $\dfrac{[A]}{[\mathrm{T}]} = [\mathrm{MLT}^{-2}] \Rightarrow [A] = [\mathrm{MLT}^{-1}]$.
$[B][\mathrm{T}^2] = [\mathrm{MLT}^{-2}] \Rightarrow [B] = [\mathrm{MLT}^{-4}]$. $[C][\mathrm{T}] = [\mathrm{MLT}^{-2}] \Rightarrow [C] = [\mathrm{MLT}^{-3}]$.

Answer: as above

Q. 18 • Combine them

For velocity $v = Ax^2 + \dfrac{B}{t^2} + \dfrac{C}{x}$, where x is displacement and t time: find the DF of A, B, C. If $\left[\dfrac{AB}{C}\right] = [\mathrm{M}^{\alpha}\mathrm{L}^{\beta}\mathrm{T}^{\gamma}]$, find $\alpha + \beta + \gamma$.

Solution. $[A][\mathrm{L}^2] = [\mathrm{LT}^{-1}] \Rightarrow [A] = [\mathrm{L}^{-1}\mathrm{T}^{-1}]$.
$\dfrac{[B]}{[\mathrm{T}^2]} = [\mathrm{LT}^{-1}] \Rightarrow [B] = [\mathrm{LT}]$. $\dfrac{[C]}{[\mathrm{L}]} = [\mathrm{LT}^{-1}] \Rightarrow [C] = [\mathrm{L}^2\mathrm{T}^{-1}]$.

$$\left[\frac{AB}{C}\right] = \frac{[\mathrm{L}^{-1}\mathrm{T}^{-1}][\mathrm{LT}]}{[\mathrm{L}^2\mathrm{T}^{-1}]} = [\mathrm{L}^{-2}\mathrm{T}]$$

So $\alpha + \beta + \gamma = 0 - 2 + 1 = -1$.

Answer: -1

Q. 19 • Product

For acceleration $a = At + \dfrac{B}{x^2} + Ct^3$, where x is displacement, find the DF of ABC.

Solution. $[A] = [\mathrm{LT}^{-3}]$, $[B] = [\mathrm{L}^3\mathrm{T}^{-2}]$, $[C] = [\mathrm{LT}^{-5}]$.
$[ABC] = [\mathrm{L}^{1+3+1}\mathrm{T}^{-3-2-5}] = [\mathrm{L}^5\mathrm{T}^{-10}]$.

Answer: $[\mathrm{L}^5\mathrm{T}^{-10}]$

Q. 20 • Inside the bracket first

(a) $F = \dfrac{Ax}{B + t^2}$, where F is force, x displacement, t time.
(b) $P = \dfrac{A}{B - t^2}$, where P is pressure.
Find the DF of A and B in each.

Solution. **(a)** B is added to t^2, so $[B] = [\mathrm{T}^2]$. Then $[\mathrm{MLT}^{-2}] = \dfrac{[A][\mathrm{L}]}{[\mathrm{T}^2]} \Rightarrow [A] = [\mathrm{M}]$.

(b) $[B] = [\mathrm{T}^2]$, and $[A] = [P][\mathrm{T}^2] = [\mathrm{ML}^{-1}]$.

Answer: (a) $[\mathrm{M}]$, $[\mathrm{T}^2]$ **(b)** $[\mathrm{ML}^{-1}]$, $[\mathrm{T}^2]$

PadhoNa Shortcut

Pehle dekho kaun kisme jud ya ghat raha hai. Sawal wahin se khulta hai. Bracket ke andar wala term sabse pehle solve karo.

Q. 21 • Fractional power

$F = \dfrac{A\sqrt{t}}{t^2 + B}$, where F is force and t is time. Find $[A]$, $[B]$ and $[AB]$.

Solution. $[B] = [\mathrm{T}^2]$. Then $[\mathrm{MLT}^{-2}] = \dfrac{[A][\mathrm{T}^{1/2}]}{[\mathrm{T}^2]} \Rightarrow [A] = [\mathrm{MLT}^{-1/2}]$.
$[AB] = [\mathrm{MLT}^{3/2}]$.

Answer: $[\mathrm{MLT}^{-1/2}]$, $[\mathrm{T}^2]$, $[\mathrm{MLT}^{3/2}]$

Q. 22 • Mixed

$\dfrac{a + A\sqrt{t}}{Bx^2} = P$, where a is acceleration, t time, x displacement, P pressure. Find the DF of A and B.

Solution. $[A][\mathrm{T}^{1/2}] = [\mathrm{LT}^{-2}] \Rightarrow [A] = [\mathrm{LT}^{-5/2}]$.
$[B] = \dfrac{[a]}{[P][x^2]} = \dfrac{[\mathrm{LT}^{-2}]}{[\mathrm{ML}^{-1}\mathrm{T}^{-2}][\mathrm{L}^2]} = [\mathrm{M}^{-1}]$.

Answer: $[\mathrm{LT}^{-5/2}]$, $[\mathrm{M}^{-1}]$

PadhoNa Active Recall #2 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. State the principle of homogeneity.
2. Is every dimensionally correct formula true? Give an example.
3. Is a dimensionless quantity always unitless? Give an example.
4. Name the four applications of dimensional analysis.
5. If $x = At^2 + Bt$ with x a length, find $[A]$ and $[B]$.
6. If $v = \dfrac{A}{B + t}$, with v velocity and t time, find $[A]$ and $[B]$.

Score: _____ / 6 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

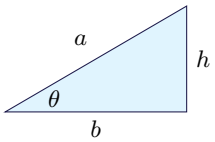
PadhoNa Test Series — **Homogeneity ke sawal = free marks**

Bracket wale sawal jitne solve karoge, utne fast honge. **PadhoNa Test Series** se apni accuracy aur speed dono check karo.

5. Dimensionless Functions

The big rule

Whatever sits **inside** $\sin$, $\cos$, $\tan$, inverse trig, $\log$, $\ln$, or in the **power of** e must be dimensionless, and the function itself is dimensionless too.



- Angle is dimensionless, since θ = arc/radius.
- All trig ratios are dimensionless: $\sin\theta = \dfrac{h}{a} = \dfrac{L}{L}$.
- In $\ln x = \sin^{-1} y$, both x and y are dimensionless.
- In $y = \log x$ and $e^x = y$, both x and y are dimensionless.
  In $a^x = y$, x is dimensionless and $[y] = [a]^x$. So y is dimensionless only if a is.

Hinglish mein samjho

$y = \sin x$ mein x ko displacement mat samajhna. x yahan ek pure number hai, aur y bhi -1 se $+1$ ke beech ka number hai. Rule simple hai: sin, cos, log, ya e ki power ke andar jitne alag terms hain, har term ko **1** ke barabar rakh do.

Q. 23 • Inside the function

Find the DF of the constants, where x is length and t is time.

(a) $y = \cos(Ax^3)$ **(b)** $y = \cos(At^2 + Bx)$ **(c)** $y = \tan\left(\dfrac{A}{t} + Bx^3\right)$

Solution. (a) $Ax^3 = 1 \Rightarrow [A] = [\mathrm{L}^{-3}]$.
(b) $At^2 = 1$ and $Bx = 1$, so $[A] = [\mathrm{T}^{-2}]$, $[B] = [\mathrm{L}^{-1}]$.
(c) $\dfrac{A}{t} = 1$ and $Bx^3 = 1$, so $[A] = [\mathrm{T}]$, $[B] = [\mathrm{L}^{-3}]$.

Answer: as above

Q. 24 • Log and exponential

(a) $y = \ln\left(\frac{A}{x^2} + Bt\right)$ **(b)** $y = e^{A/x + Bt^2}$. Here x is length and t is time. Find the DF of A and B.

Solution. **(a)** $[A] = [\mathrm{L}^2]$, $[B] = [\mathrm{T}^{-1}]$. **(b)** $[A] = [\mathrm{L}]$, $[B] = [\mathrm{T}^{-2}]$.

Answer: as above

Q. 25 • With an amplitude

$y = A\cos(Bx + C)$, where A is an amplitude in metres and x is a length. Find the DF of B, C and y.

Solution. $Bx = 1 \Rightarrow [B] = [\mathrm{L}^{-1}]$. C is added inside, so it is dimensionless. The cosine is dimensionless, so $[y] = [A] = [\mathrm{L}]$.

Answer: $[\mathrm{L}^{-1}]$, dimensionless, $[\mathrm{L}]$

Q. 26 • Length and acceleration inside

$y = A\sin\left(\frac{B}{x} + Ca\right)$, where x is a length and a is acceleration. Find the DF of B and C.

Solution. $\frac{B}{x} = 1 \Rightarrow [B] = [\mathrm{L}]$. $Ca = 1 \Rightarrow [C] = [\mathrm{L}^{-1}\mathrm{T}^2]$.

Answer: $[\mathrm{L}]$, $[\mathrm{L}^{-1}\mathrm{T}^2]$

Q. 27 • Radioactive decay

$N = N_0 e^{-\lambda t}$. Find the DF of λ.

Solution. λt is dimensionless, so $[\lambda] = [\mathrm{T}^{-1}]$.

Answer: $[\mathrm{T}^{-1}]$

Q. 28 • Three terms in the power

$y = Ae^{Bt^2 + Ct + Dx^3}$, where y and x are lengths and t is time. Find the DF of A, B, C and D.

Solution. $[A] = [y] = [\mathrm{L}]$. $[B] = [\mathrm{T}^{-2}]$, $[C] = [\mathrm{T}^{-1}]$, $[D] = [\mathrm{L}^{-3}]$.

Answer: as above

Q. 29 • Boltzmann in the power

$E = \frac{\alpha}{\beta} e^{-\alpha z / k\theta}$, where E is energy, z distance, k Boltzmann constant, θ temperature. Find the DF of α and β.

Solution. $k\theta$ has the DF of energy. So $[\alpha][\mathrm{L}] = [\mathrm{ML}^2\mathrm{T}^{-2}] \Rightarrow [\alpha] = [\mathrm{MLT}^{-2}]$.
$[E] = \frac{[\alpha]}{[\beta]} \Rightarrow [\beta] = \frac{[\mathrm{MLT}^{-2}]}{[\mathrm{ML}^2\mathrm{T}^{-2}]} = [\mathrm{L}^{-1}]$.

Answer: $[\mathrm{MLT}^{-2}]$, $[\mathrm{L}^{-1}]$

Q. 30 • Velocity with time

$v(t) = \frac{a_0}{\beta}(1 - e^{-\beta t})$. Find the DF of a_0 and β.

Solution. βt is dimensionless, so $[\beta] = [\mathrm{T}^{-1}]$. The bracket is dimensionless, so $[a_0] = [v][\beta] = [\mathrm{LT}^{-2}]$.

Answer: $[\mathrm{LT}^{-2}]$, $[\mathrm{T}^{-1}]$

PadhoNa Active Recall #3 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. List the functions whose argument must be dimensionless.
2. If $y = \sin(At + Bx)$, with t time and x length, find $[A]$ and $[B]$.
3. Find the DF of RC-type constant τ in $q = q_0 e^{-t/\tau}$.
4. In $F = F_0 e^{-kx}$, find $[k]$ for x in metres.

Score: _____ / 4 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series — **Sin ke andar ka khel samajh aaya?**
Ab bina dekhe 10 sawal lagao. **PadhoNa Test Series** mein aise conceptual sawal time ke saath milenge.

6. Deriving Relations Between Quantities

Method

1. Assume $Q = k\,A^x B^y C^z$, where k is a dimensionless constant.
2. Write the DF of each quantity.
3. Compare the powers of M, L and T on both sides.
4. Solve for x, y, z and write the formula.

Hinglish mein samjho

Sawal ki physics abhi samajhna zaroori nahi. Physics do line ki hai, maths das line ki ho sakti hai. Bas powers compare karo aur equations dhyan se solve karo. Saare sawal ek hi pattern ke hain.

Q. 31 • Mass on a spring

The period T of a mass on a spring depends on the mass m and spring constant k_s. Find the formula.

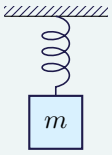
Solution.

Let $T = k\, m^x k_s^y$, with $[k_s] = [\mathrm{MT^{-2}}]$.

$$[\mathrm{T}] = [\mathrm{M}]^x[\mathrm{MT^{-2}}]^y = [\mathrm{M^{x+y}T^{-2y}}]$$

M: $x + y = 0$. T: $-2y = 1$. So $y = -\frac{1}{2}$, $x = \frac{1}{2}$.

$$T = k\sqrt{\frac{m}{k_s}}$$



The true formula has $k = 2\pi$. Dimensions cannot give that number.

Answer: $T = k\sqrt{m/k_s}$

Q. 32 • Vibrating string

The frequency f of a stretched string depends on its length l, tension F and mass per unit length μ. Find the formula.

Solution.

Let $f = k\, l^a F^b \mu^c$, with $[\mu] = [\mathrm{ML^{-1}}]$.

$$[\mathrm{T^{-1}}] = [\mathrm{L}]^a[\mathrm{MLT^{-2}}]^b[\mathrm{ML^{-1}}]^c$$

M: $b + c = 0$. L: $a + b - c = 0$. T: $-2b = -1$.
So $b = \frac{1}{2}$, $c = -\frac{1}{2}$, $a = -1$.

$$f = \frac{k}{l}\sqrt{\frac{F}{\mu}}$$



Answer: $f = \frac{k}{l}\sqrt{F/\mu}$

Q. 33 • Bubble oscillation

A gas bubble from an underwater explosion oscillates with a period T that depends on pressure P, density d and energy E. Find the relation.

Solution. Let $T = k\, P^x d^y E^z$:

$$[\mathrm{T}] = [\mathrm{ML^{-1}T^{-2}}]^x[\mathrm{ML^{-3}}]^y[\mathrm{ML^2T^{-2}}]^z$$

M: $x + y + z = 0$. L: $-x - 3y + 2z = 0$. T: $-2x - 2z = 1$.
From T: $x + z = -\frac{1}{2}$. Then M gives $y = \frac{1}{2}$. Then L gives $-x + 2z = \frac{3}{2}$.
Solving with $x + z = -\frac{1}{2}$: $z = \frac{1}{3}$, $x = -\frac{5}{6}$.

$$T = k\, P^{-5/6} d^{1/2} E^{1/3}$$

Answer: $T = kP^{-5/6}d^{1/2}E^{1/3}$

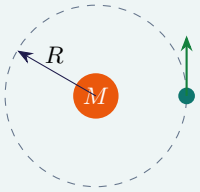
Q. 34 • Planet in orbit

The period T of a planet depends on the orbit radius R, the mass of the Sun M and G. Find the formula.

Solution.

Let $T = k\,R^a M^b G^c$:

$$[\mathrm{T}] = [\mathrm{L}]^a[\mathrm{M}]^b[\mathrm{M^{-1}L^3T^{-2}}]^c$$



T: $-2c = 1 \Rightarrow c = -\frac{1}{2}$.
M: $b - c = 0 \Rightarrow b = -\frac{1}{2}$.
L: $a + 3c = 0 \Rightarrow a = \frac{3}{2}$.

$$T = k\sqrt{\frac{R^3}{GM}}$$

Answer: $T = k\sqrt{R^3/GM}$

Constant k ka sach

In sawalon mein k ko hamesha dimensionless maante hain. Agar k ki apni dimension ho (jaise G ya R), to yeh method kaam nahi karega. Yeh dimension analysis ki ek badi limitation hai.

Q. 35 • Sum of powers

The centripetal force on a body is $F = k\,m^x v^y r^z$, where k is dimensionless. Find $x + y + z$.

Solution. $[\mathrm{MLT^{-2}}] = [\mathrm{M}]^x[\mathrm{LT^{-1}}]^y[\mathrm{L}]^z$. M: $x = 1$. T: $-y = -2$, so $y = 2$. L: $y + z = 1$, so $z = -1$.
$x + y + z = 2$.

Answer: 2

Q. 36 • Falling raindrop

A small raindrop falling through air feels a drag force F that depends on the viscosity η, its radius r and speed v. Find the form of F.

Solution. Let $F = k\,\eta^a r^b v^c$: $[\mathrm{MLT^{-2}}] = [\mathrm{ML^{-1}T^{-1}}]^a[\mathrm{L}]^b[\mathrm{LT^{-1}}]^c$.
M: $a = 1$. T: $-a - c = -2 \Rightarrow c = 1$. L: $-a + b + c = 1 \Rightarrow b = 1$.

Answer: $F = k\eta r v$

Q. 37 • Liquid in a tube

The volume per second Q of a liquid through a narrow tube depends on the pressure gradient $G_p = P/l$, the tube radius r and the viscosity η. Find the formula.

Solution. Let $Q = k\, G_p^a r^b \eta^c$, with $[G_p] = [\mathrm{ML^{-2}T^{-2}}]$:

$$[\mathrm{L^3T^{-1}}] = [\mathrm{ML^{-2}T^{-2}}]^a[\mathrm{L}]^b[\mathrm{ML^{-1}T^{-1}}]^c$$

M: $a + c = 0$. T: $-2a - c = -1$. So $a = 1, c = -1$. L: $-2a + b - c = 3 \Rightarrow b = 4$.

$$Q = k\frac{Pr^4}{\eta l}$$

Answer: $Q = kPr^4/(\eta l)$

Q. 38 • Liquid pressure

The pressure at depth h in a liquid depends on h, the density ρ and gravity g. Find the formula.

Solution. Let $P = k\,\rho^a g^b h^c$: $[\mathrm{ML^{-1}T^{-2}}] = [\mathrm{ML^{-3}}]^a[\mathrm{LT^{-2}}]^b[\mathrm{L}]^c$.
M: $a = 1$. T: $-2b = -2$, so $b = 1$. L: $-3a + b + c = -1 \Rightarrow c = 1$.

Answer: $P = k\rho gh$

Limitations of dimensional analysis

- It cannot find dimensionless constants such as 2π or $\frac{1}{2}$.
- It fails when the constant has dimensions of its own.
- It works only for product-of-powers formulas. It cannot derive $s = ut + \frac{1}{2}at^2$ or $y = A\sin\omega t$.
- With M, L, T alone, it can handle at most three unknown quantities.

PadhoNa Active Recall #4

Notes band karo. Yaad se likho.

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. What do we assume about k while deriving relations?
2. Write the DF of mass per unit length and of pressure gradient.
3. Can dimensional analysis derive $s = ut + \frac{1}{2}at^2$? Why?
4. Time period of a pendulum depends on l and g. Derive it.
5. State one limitation of dimensional analysis.

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

7. Unit Conversion

Conversion formula

If a quantity has the DF $[\mathrm{M^aL^bT^c}]$, then from $n_1u_1 = n_2u_2$:

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

PadhoNa Shortcut

Chhota tareeka: unit ko seedha todo. $1\,\mathrm{kg} = 1000\,\mathrm{g}$, $1\,\mathrm{m} = 100\,\mathrm{cm}$ likho aur simplify karo. Dono methods ka answer same aata hai.

Q. 39 • Newton and dyne

Convert $1\,\mathrm{N}$ into dyne.

Solution. **Method 1:** force is $[\mathrm{MLT^{-2}}]$, so $n_2 = 1\left[\frac{\mathrm{kg}}{\mathrm{g}}\right]^1\left[\frac{\mathrm{m}}{\mathrm{cm}}\right]^1\left[\frac{\mathrm{s}}{\mathrm{s}}\right]^{-2} = 1000 \times 100 = 10^5$.

Method 2: $1\,\mathrm{N} = \frac{1\,\mathrm{kg\,m}}{\mathrm{s^2}} = \frac{(1000\,\mathrm{g})(100\,\mathrm{cm})}{\mathrm{s^2}} = 10^5\,\mathrm{dyne}$.

Answer: 10^5 dyne

Q. 40 • Joule and erg

Convert $5\,\mathrm{J}$ into erg.

Solution. $1\,\mathrm{J} = \frac{(1000\,\mathrm{g})(100\,\mathrm{cm})^2}{\mathrm{s^2}} = 10^7\,\mathrm{erg}$. So $5\,\mathrm{J} = 5 \times 10^7\,\mathrm{erg}$.

Answer: 5×10^7 erg

Q. 41 • Viscosity units

$[\eta] = [\mathrm{ML^{-1}T^{-1}}]$. How many poise (CGS unit) equal 1 poiseuille (SI unit, kg $\mathrm{m^{-1}\,s^{-1}}$)?

Solution. $1\,\mathrm{kg\,m^{-1}\,s^{-1}} = (1000\,\mathrm{g})(100\,\mathrm{cm})^{-1}\mathrm{s^{-1}} = 10\,\mathrm{g\,cm^{-1}\,s^{-1}} = 10$ poise.

Answer: 10 poise

Q. 42 • Value of G

$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$ in SI. Find its value in CGS.

Solution. $[G] = [\mathrm{M^{-1}L^3T^{-2}}]$. So $n_2 = 6.67 \times 10^{-11}(10^3)^{-1}(10^2)^3 = 6.67 \times 10^{-8}\,\mathrm{dyne\,cm^2\,g^{-2}}$.

Answer: 6.67×10^{-8} CGS

Q. 43 • New system

In a new system, the unit of mass is α kg, of length β m, and of time γ s. Find the value of atmospheric pressure, 1.0×10^5 Pa, in this system.

Solution. $[P] = [\mathrm{ML^{-1}T^{-2}}]$, so

$$n_2 = 10^5 \left[\frac{1}{\alpha}\right]^1 \left[\frac{1}{\beta}\right]^{-1} \left[\frac{1}{\gamma}\right]^{-2} = 10^5\, \alpha^{-1}\beta\gamma^2$$

Answer: $10^5\alpha^{-1}\beta\gamma^2$

Hinglish mein samjho

Hypothetical unit ke sawalon mein ulta mat ho jana. Pehle nikalo "naya 1 unit kitne SI ke barabar hai". Phir jo value chahiye use us naye unit se divide kar do.

Q. 44 • Hypothetical force

In a system, 1 unit of mass = 3 kg, 1 unit of length = 4 m, 1 unit of time = 2 s. Find the value of the new unit of force in N, and how many new units equal 60 N.

Solution. $1\,\mathrm{N'} = \dfrac{(3\,\mathrm{kg})(4\,\mathrm{m})}{(2\,\mathrm{s})^2} = 3\,\mathrm{N}$. So $1\,\mathrm{N} = \dfrac{1}{3}\,\mathrm{N'}$, and $60\,\mathrm{N} = 20\,\mathrm{N'}$.

Answer: 3 N; 20 N′

Q. 45 • Another system

1 unit of mass = 10 kg, 1 unit of length = 2 m, 1 unit of time = 5 s. Find **(i)** 1 unit of force in N; **(ii)** the value of 1 N in this system; **(iii)** the value of 40 N in this system.

Solution. **(i)** $1\,\mathrm{N'} = \dfrac{10 \times 2}{25} = 0.8\,\mathrm{N}$. **(ii)** $1\,\mathrm{N} = \dfrac{1}{0.8} = 1.25\,\mathrm{N'}$. **(iii)** $40 \times 1.25 = 50\,\mathrm{N'}$.

Answer: 0.8 N; 1.25; 50

Q. 46 • Energy in a new system

1 unit of mass = 5 kg, 1 unit of length = 4 m, 1 unit of time = 2 s. Find **(1)** 1 unit of force in N; **(2)** 1 unit of energy in J; **(3)** the value of 300 J in this system.

Solution. **(1)** $\dfrac{5 \times 4}{4} = 5\,\mathrm{N}$. **(2)** $\dfrac{5 \times 16}{4} = 20\,\mathrm{J}$. **(3)** $\dfrac{300}{20} = 15$ units.

Check by formula: $n_1 M_1 L_1^2 T_1^{-2} = n_2 M_2 L_2^2 T_2^{-2} \Rightarrow 300 = n_2 \times \dfrac{5 \times 16}{4} \Rightarrow n_2 = 15$ ✓

Answer: 5 N; 20 J; 15

PadhoNa Active Recall #5 — **Notes band karo. Yaad se likho.**

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Write the conversion formula for n_2.
2. How many dyne in 1 N, and erg in 1 J?
3. How many poise in 1 poiseuille?
4. Unit of mass 2 kg, length 3 m, time 1 s. Find the unit of force.
5. In that same system, how many units is 24 N?

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series — **Conversion mein galti = seedha minus**

Power ka sign ek baar galat hua to answer gaya. **PadhoNa Test Series** mein practice karo, taaki exam mein aisi galti na ho.

8. Must-Solve Mixed Problems

Hinglish mein samjho

Yeh sawal exam ke level ke hain. Pen uthao aur pehle khud try karo. Har sawal ek hi cheez poochh raha hai: DF sahi likho aur powers compare karo.

Q. 47 • Dimension of velocity

If Y is Young's modulus and ρ is density, express the dimension of velocity in terms of Y and ρ.

Solution. **Method 1:** $\dfrac{[Y]}{[\rho]} = \dfrac{[\mathrm{ML^{-1}T^{-2}}]}{[\mathrm{ML^{-3}}]} = [\mathrm{L^2T^{-2}}]$. So $[v] = [Y^{1/2}\rho^{-1/2}]$.

Method 2: Let $v = kY^a\rho^b$. Then M: $a + b = 0$, T: $-2a = -1$. So $a = \frac{1}{2}$, $b = -\frac{1}{2}$.

Answer: $[Y^{1/2}\rho^{-1/2}]$

Q. 48 • Wave on a string

The speed v of a wave on a string depends on tension F and mass per unit length μ. Find the relation.

Solution. Let $v = kF^a\mu^b$: $[\mathrm{LT^{-1}}] = [\mathrm{MLT^{-2}}]^a[\mathrm{ML^{-1}}]^b$.
M: $a + b = 0$. T: $-2a = -1$. So $a = \frac{1}{2}$, $b = -\frac{1}{2}$. L check: $a - b = 1$ ✓

Answer: $v = k\sqrt{F/\mu}$

Q. 49 • Ripples

The speed v of short ripples on water depends on surface tension S, density ρ and wavelength λ. Find the relation.

Solution. Let $v = kS^a\rho^b\lambda^c$: $[\mathrm{LT^{-1}}] = [\mathrm{MT^{-2}}]^a[\mathrm{ML^{-3}}]^b[\mathrm{L}]^c$.
T: $-2a = -1 \Rightarrow a = \frac{1}{2}$. M: $a + b = 0 \Rightarrow b = -\frac{1}{2}$. L: $-3b + c = 1 \Rightarrow c = -\frac{1}{2}$.

Answer: $v = k\sqrt{S/(\rho\lambda)}$

Q. 50 • Energy density

If μ_0 is the permeability of free space and B is a magnetic field, find the dimensions of $\dfrac{B^2}{\mu_0}$.

Solution. **Hint:** magnetic energy density $= \dfrac{B^2}{2\mu_0}$, so the DF is that of energy per volume, which is pressure.

Check: $\dfrac{[\mathrm{MT^{-2}A^{-1}}]^2}{[\mathrm{MLT^{-2}A^{-2}}]} = [\mathrm{ML^{-1}T^{-2}}]$ ✓

Answer: $[\mathrm{ML^{-1}T^{-2}}]$

PadhoNa Shortcut

$\dfrac{1}{2}\varepsilon_0 E^2$ aur $\dfrac{B^2}{2\mu_0}$ dono energy density hain. Dono ki DF pressure jaisi hoti hai: $[\mathrm{ML^{-1}T^{-2}}]$.

Q. 51 • Match the lists

Match List-I with List-II.

List-I		List-II	
A	Planck's constant	I	$[\mathrm{ML^2T^{-3}}]$
B	Power	II	$[\mathrm{MLT^{-1}}]$
C	Spring constant	III	$[\mathrm{ML^2T^{-1}}]$
D	Impulse	IV	$[\mathrm{MT^{-2}}]$

Solution. A: $E = h\nu$, so $[h] = [\mathrm{ML^2T^{-1}}]$: III. B: energy/time: I. C: force/length: IV. D: force $\times$ time: II.

Answer: A-III, B-I, C-IV, D-II

Q. 52 • Real gas

For 1 mole of a real gas, the van der Waals equation is $\left(P + \dfrac{a}{V^2}\right)(V - b) = RT$. Taking V as the volume of that 1 mole and ignoring the mole dimension, what quantity has the same dimensions as $\dfrac{a}{b}$?

Solution. $[a] = [PV^2]$ and $[b] = [V]$. So $\left[\dfrac{a}{b}\right] = [PV] = [\mathrm{ML^{-1}T^{-2}}][\mathrm{L^3}] = [\mathrm{ML^2T^{-2}}]$, which is energy.

Answer: Energy

Galti mat karna

$(P + a/V^2)$ wale bracket mein a/V^2 ki dimension pressure ki hai, aur $(V - b)$ mein b ki volume ki. Yehi homogeneity ka sabse famous example hai.

Q. 53 • Planck mass

A mass m is built from c, h and G as $m = c^{\alpha}h^{\beta}G^{\gamma}$. Find α, β, γ.

Solution. $[\mathrm{M}] = [\mathrm{LT}^{-1}]^{\alpha}[\mathrm{ML}^2\mathrm{T}^{-1}]^{\beta}[\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}]^{\gamma}$.
M: $\beta - \gamma = 1$. L: $\alpha + 2\beta + 3\gamma = 0$. T: $-\alpha - \beta - 2\gamma = 0$.
Adding L and T: $\beta + \gamma = 0$. So $\beta = \frac{1}{2}$, $\gamma = -\frac{1}{2}$, and then $\alpha = \frac{1}{2}$.

Answer: $\alpha = \frac{1}{2},\ \beta = \frac{1}{2},\ \gamma = -\frac{1}{2}$

Q. 54 • Electric field from constants

In some system, the electric field is expressed by the electron charge e, electron mass m_e, Planck's constant h and Coulomb's constant $k = \dfrac{1}{4\pi\varepsilon_0}$, as $[E] = [e]^{\alpha}[m_e]^{\beta}[h]^{\gamma}[k]^{\delta}$. Find $\alpha + \beta + \gamma + \delta$.

Solution. Using $[e] = [\mathrm{AT}]$, $[h] = [\mathrm{ML}^2\mathrm{T}^{-1}]$, $[k] = [\mathrm{ML}^3\mathrm{T}^{-4}\mathrm{A}^{-2}]$ and $[E] = [\mathrm{MLT}^{-3}\mathrm{A}^{-1}]$:
M: $\beta + \gamma + \delta = 1$. L: $2\gamma + 3\delta = 1$. T: $\alpha - \gamma - 4\delta = -3$. A: $\alpha - 2\delta = -1$.
From A: $\alpha = 2\delta - 1$. Into T: $\gamma = 2 - 2\delta$. Into L: $4 - 4\delta + 3\delta = 1 \Rightarrow \delta = 3$.
So $\gamma = -4$, $\alpha = 5$, $\beta = 2$. Sum $= 5 + 2 - 4 + 3 = 6$.

Answer: 6

Q. 55 • New fundamental quantities

If velocity v, acceleration a and force F are taken as fundamental, find the dimensions of mass, length and time.

Solution. $F = ma \Rightarrow$ mass $= [Fa^{-1}]$.
$v = at \Rightarrow$ time $= [va^{-1}]$.
$v^2 = 2as \Rightarrow$ length $= [v^2a^{-1}]$.

Answer: $[Fa^{-1}]$, $[v^2a^{-1}]$, $[va^{-1}]$

Q. 56 • Exponential energy

The energy of a molecule is $E = \alpha\beta^2 e^{-\beta x^2/kT}$, where x is distance, k is the Boltzmann constant and T is temperature. Find the dimensions of α and β.

Solution. $[\beta x^2] = [kT] = [\mathrm{ML}^2\mathrm{T}^{-2}] \Rightarrow [\beta] = [\mathrm{MT}^{-2}]$.
$[\alpha][\beta]^2 = [\mathrm{ML}^2\mathrm{T}^{-2}] \Rightarrow [\alpha] = \dfrac{[\mathrm{ML}^2\mathrm{T}^{-2}]}{[\mathrm{M}^2\mathrm{T}^{-4}]} = [\mathrm{M}^{-1}\mathrm{L}^2\mathrm{T}^2]$.

Answer: $[\mathrm{M}^{-1}\mathrm{L}^2\mathrm{T}^2]$, $[\mathrm{MT}^{-2}]$

Q. 57 • Strange system

In a system of units, mass and velocity are both dimensionless, and length has the dimension L. Which are correct?
(A) Dimension of time is L. (B) Dimension of force is L^{-1}. (C) Energy is dimensionless. (D) Dimension of power is L^{-2}. (E) Linear momentum has dimension L.

Solution. Velocity is dimensionless, so $\frac{L}{T} = 1$ and $T = L$: (A) is true.
Force $= \frac{\text{mass} \times \text{velocity}}{\text{time}} = L^{-1}$: (B) is true.
Energy $=$ mass $\times$ velocity^2: dimensionless, so (C) is true.
Power $= \frac{\text{energy}}{\text{time}} = L^{-1}$: (D) is false.
Momentum $=$ mass $\times$ velocity: dimensionless, so (E) is false.

Answer: (A), (B), (C)

PadhoNa Active Recall #6

Notes band karo. Yaad se likho.

Rukko! Peeche page mat palatna. Pen uthao aur har sawal copy mein khud se solve karo. Jo atak gaya, wahi tumhara asli revision topic hai.

1. Give the DF of $\varepsilon_0 E^2$.
2. For 1 mole of gas, in $(P + a/V^2)(V - b) = RT$, give the DF of a and b.
3. Mass, length, time are replaced by F, v, t. Give the DF of mass.
4. $[h]$ matches which mechanical quantity?
5. In a system with dimensionless mass and velocity, find the dimension of power.

Score: _____ / 5 Answers last ke Answer Key mein check karo. Pehle attempt, phir check!

PadhoNa Test Series

Poora chapter complete. Ab test do!

Agar tumne har sawal khud solve kiya to bahut badhiya. Ab **PadhoNa Test Series** ka chapter test do aur weak topics ko notes se dobara revise karo.

9. Master Formula Sheet

Core ideas

$Q = n_1 u_1 = n_2 u_2$, so $n \propto \frac{1}{u}$

Added terms have the same DF

Inside sin, log, e : dimensionless

$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

Mechanics

Velocity $[\mathrm{LT}^{-1}]$ Acceleration $[\mathrm{LT}^{-2}]$

Force $[\mathrm{MLT}^{-2}]$ Momentum $[\mathrm{MLT}^{-1}]$

Energy $[\mathrm{ML}^2\mathrm{T}^{-2}]$ Power $[\mathrm{ML}^2\mathrm{T}^{-3}]$

Pressure $[\mathrm{ML}^{-1}\mathrm{T}^{-2}]$ Density $[\mathrm{ML}^{-3}]$

Ang. momentum $[\mathrm{ML}^2\mathrm{T}^{-1}]$ G $[\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}]$

Surface tension $[\mathrm{MT}^{-2}]$ Viscosity $[\mathrm{ML}^{-1}\mathrm{T}^{-1}]$

Heat

Specific heat $[\mathrm{L}^2\mathrm{T}^{-2}\mathrm{K}^{-1}]$ Latent heat $[\mathrm{L}^2\mathrm{T}^{-2}]$

k_B $[\mathrm{ML}^2\mathrm{T}^{-2}\mathrm{K}^{-1}]$ R $[\mathrm{ML}^2\mathrm{T}^{-2}\mathrm{K}^{-1}\mathrm{mol}^{-1}]$

Electricity

Charge $[\mathrm{AT}]$ Voltage $[\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-1}]$

Resistance $[\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-2}]$

Capacitance $[\mathrm{M}^{-1}\mathrm{L}^{-2}\mathrm{T}^4\mathrm{A}^2]$

Inductance $[\mathrm{ML}^2\mathrm{T}^{-2}\mathrm{A}^{-2}]$

Magnetic field $[\mathrm{MT}^{-2}\mathrm{A}^{-1}]$

Unit facts

$1\,\mathrm{N} = 10^5$ dyne, $1\,\mathrm{J} = 10^7$ erg

1 poiseuille = 10 poise

$1\,\mathrm{g/cm^3} = 1000\,\mathrm{kg/m^3}$

10. Active Recall Answer Key

Pehle khud attempt kiya? Tabhi yeh page dekho. Jo galat hua, us topic par wapas jao.

Active Recall #1

1. Length m, mass kg, time s, temperature K, current A, luminous intensity cd, amount of substance mol. **2.** It becomes $1/4$. **3.** $[\mathrm{MLT^{-2}}]$, $[\mathrm{ML^2T^{-2}}]$, $[\mathrm{ML^2T^{-3}}]$. **4.** $[\mathrm{MLT^{-1}}]$, $[\mathrm{ML^2T^{-1}}]$. **5.** 2000. **6.** Work or energy. **7.** $[\mathrm{AT}]$; $[\mathrm{ML^2T^{-3}A^{-1}}]$.

Active Recall #2

1. All added, subtracted or equated terms must have the same dimensions. **2.** No; $K = mv^2$. **3.** No; plane angle in radians. **4.** Find a DF, check a formula, derive a relation, convert units. **5.** $[\mathrm{LT^{-2}}]$, $[\mathrm{LT^{-1}}]$. **6.** $[\mathrm{L}]$, $[\mathrm{T}]$.

Active Recall #3

1. Trig, inverse trig, log, ln, and powers of e. **2.** $[\mathrm{T^{-1}}]$, $[\mathrm{L^{-1}}]$. **3.** $[\mathrm{T}]$. **4.** $[\mathrm{L^{-1}}]$.

Active Recall #4

1. It is dimensionless. **2.** $[\mathrm{ML^{-1}}]$, $[\mathrm{ML^{-2}T^{-2}}]$. **3.** No, it has added terms, so it is not a single product of powers. **4.** $T = k\sqrt{l/g}$. **5.** It cannot give dimensionless constants.

Active Recall #5

1. $n_2 = n_1[M_1/M_2]^a[L_1/L_2]^b[T_1/T_2]^c$. **2.** 10^5; 10^7. **3.** 10. **4.** 6 N. **5.** 4.

Active Recall #6

1. $[\mathrm{ML^{-1}T^{-2}}]$. **2.** $[\mathrm{ML^5T^{-2}}]$, $[\mathrm{L^3}]$. **3.** $[Fv^{-1}t]$. **4.** Angular momentum. **5.** L^{-1}.

Chapter khatam. Ab shuru hota hai asli game!

Notes se samajh aati hai. Test se confidence aata hai. Apni speed, accuracy aur weak topics jaanne ke liye **PadhoNa Test Series** abhi join karo aur har chapter ke baad khud ko check karo.

Padho samajh ke. Yaad karo khud se. Test do roz se.